IMPROVING PUPILS GEOMETRIC THINKING THROUGH VAN HIELE'S PHASE-BASED LEARNING USING TANGRAM PUZZULE

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Abstract

The study determined pupils' performance in geometric thinking through Van Hiele's Phase –based learning using tangram puzzle. The study was carried out in Department of Primary Education Studies demonstration school of Alvan lkoku University of Education in Owerri Municipal Council Area of Imo-State. Based on the Objectives of the study, four research questions and three hypotheses were posed for the study. The design used for the study was one-group pretest-posttest design. A sample of 70 primary six (6) pupils' involving 40 males and 30 female's pupils were used for the study. After pre-test, the same group of pupils was exposed through Van Hiele's phase –based learning using tangrams. The instrument used for data collection was a 20 multiple choice test items constructed by the researchers. It had reliability coefficient(r) of 0.87 determined using Pearson product moment method .The data generated was analyzed using mean for the research questions and ANCOVA were used to test the hypotheses at 0.05 level of significant. The result showed that the use of tangram puzzle as in-class activity through Van Hiele's five phase of learning effectively improved primary six pupils in their geometric thinking, low ability pupils were observed to have greatest improvement score compared to medium and high ability pupils in geometric thinking and the use of tangram *puzzle through van Hieles five- phase of learning improved pupils achievement in geometric thinking irrespective of gender. Some useful recommendations made that tangram puzzle should be used in teaching geometry in primary schools. Primary school mathematics teachers should be trained through intensive seminars, workshops and in-service trainings on the use of manipulative for teaching and learning of mathematics and Government should established mathematics laboratories in primary schools.*

Keywords: Geometric Thinking, Van Hieles phase-learning, Trangram puzzle and Mathematics.

The world is a global village and this has made science and technology very essential in every nation. Mathematics being the queen of science and technology cannot be left behind, hence the need to acquire the knowledge of mathematics all over the world. Mathematics is not only vital in science and technology but very essential in everyday living and in various disciplines. On account of its importance in daily living and in the national economy, Nigeria government has made mathematics compulsory as a core subject in basic and secondary education. Irrespective of the great importance of mathematics in nation building, scientific and technological development, it is notable that the students' performance in mathematics at internal and external examinations has remained considerably poor (odill 2005). Ekwueme and Ali (2003) stated that, despite the important position mathematics occupies; it still remains one of the subjects that students persistently perform poorly in. Researchers like Harbor- Peters (2001), had discovered use of inappropriate teaching method as one of the factors responsible for such incessant failure in mathematics. Odill (2005) had recommended that the use of teaching methods that are practical, learner-centered, activity-based and meaningful to the needs of the learner might result in better performance in mathematics (National council for Teachers Mathematics, 2000, West African examination Council, 2006). Mathematics as a subject prepares an individual for critical thinking which is required for problem solving in the society. Ado (2012) stated that mathematics is a way of thinking that enables a learner to identity patterns and structures in order to solve problems. Mathematics is distributed into many branches which include trigonometry, algebra, construction, sets circles and geometry etc. Geometry is an important part of mathematics curriculum. It is the study of both linear measurement and angular/rotational measurement. It is also the study of shapes and figures bounded by lines and circles (NCTM, 2000).

Geometry helps students gain basic skills such as analysis, comparison, and generalization and is useful in representing and solving problems in other areas of mathematics and in real world situations. Geometry is one aspect of mathematics that creates anxiety and phobia among students. According to Idris (2007) difficulties in learning geometry among student could be explained in terms of poor cognitive development. Zanzah, 2000, Kamina and Lyer, 2009 describe inability of students to engage actively in the learning process as a factor. The need for improved achievement in mathematics has driven teachers and researchers to seek appropriate instructional strategies. These instructional strategies are the ones that will allow students to control their learning process and as well develop the required interest in mathematics. According to Jegede, Alaymola and Okebukola (1990), the increasing awareness of the importance of learner-centeredness in the teaching-learning situation has generated a lot of attention in relation to understanding how learners learn and how to help them learn about concepts. These efforts in assisting learners to learn more effectively has led to the development of meta-cognitive strategies to enhance meaningful learning (Biggs' 1998', Chiburn; 1990). According to Novak (1987) Meta - cognitive strategies are strategies that enhance the learner to take charge of his/her own learning in a highly meaningful way. Borich (2004) noted that Meta cognitive strategy which is a strategy used in self directed learning are mental processes that assist learners to reflect on their thinking by internalizing, understanding, and recalling the content to be learned. This recommendation seems to call for an option of using Van Hiele's Phases-based learning. Van Hiele model is designed for geometry learning activities (Breyfogle & Lynch, 2010; Erdogan , Akkaya , & celebo 2009).

The model consists of five levels of geometric understanding. These levels are visualization, analysis, informal deduction, formal deduction, and rigor. According to the first (visual) level of Van Hiele (1986's) geometric model of thinking, learners visually recognize shapes and figures by their global appearance for example, learners recognize triangles, squares, parallelograms and so forth by their shape, but they do not explicitly identify the properties of these figures. At the second (Analysis) level, learners start analyzing the properties of figures and learn the appropriate technical terminology for describing them but they do not interrelate. At the third (informal deduction) level, learners can identity relationship between classes of figures and discover properties of classes of figures by simple logical deduction for example, a square is considered as a rectangle because it has all the properties of a rectangle. Spear (1993) postulated that the first three levels are within the capacity of primary school learners. These learners at the primary school level should at least attain the first two Van Hiele's level in order to move effectively from one level of thinking to another. The Van Hiele's theory stresses the use of hands-on-manipulative in teaching geometry to facilitate the transition from one level of thinking to the next (Fuys, Geddes, Tischler 1988). Research on the teaching and learning of geometry also indicate that physical experience, especially the physical manipulation of geometric shapes are necessary in order for students to gain a firm understanding of geometric relationships and manipulative teaching and learning resources have much to offer when properly used (Tehoshanov 2011).Manipulative teaching and learning resources are physical objects that can be touched, turned, rearranged and collected (Brown, 2007). In order words, manipulative objects are physical objects that appeal to several of the sense where students are able to see, touch, handle and move. A manipulative material helps children in bridging their concrete sensory environment to the abstract understanding of mathematics (Bayram, 2004; Trespalacios, 2008; Ojose and Sexton, 2009). Singh (2004) opined that one of the manipulative objects that enhance student's geometric thinking and reasoning process is known as tangram puzzle.

A tangram is the oldest chiness puzzle that consists of seven geometric pieces of shapes, called tans (Tian, 2012). The seven pieces include a square, a parallelogram, too big right triangles, medium size right triangles and two small right triangles. The three basic shapes consist of a triangle, a square and parallelogram, which fit together in various ways to form polygons such as a large square, rectangle or triangle. Tangrams allow children to develop geometric concepts by categorizing, comparing and working out the puzzle and there upon to solve problems in geometric contents. When children touch and manipulate concrete objects, they become more proficient in knowing positions or locations in space and structure. Ultimately, hands-on investigation of geometric objects helps young children develop a strong intuitive grasp of geometric properties and relationships (NJMCF, 1995). Accordingly, learning geometry with tangram can help children develop their skills of geometry vocabulary, shape identification, shape orientation and discover relationship between and among the geometric shapes (Bohoning and Althouse, 1997; NCTM, 2003).Studies show that tangram inspire children's observation, imagination, shape analysis, creativity and logical thinking (Olkun, Altun and Smith 2005, Yang and Chen, 2010). Ainsa (1999) observed no significant difference between manipulatives and physical manipulatives in color matching, number matching, shape identification, counting ability, or addition/subtraction among respondents. Suh and Moyer (2007) examined the effect of developing students' representational fluency using manipulatives. The results showed that although the different manipulative models had different features, that maninpulatives were effective in supporting students' learning and encouraging relational thinking and algebraic reasoning. Reimer and Moyer (2005) also investigated the impact of manipulatives on the instruction of equivalent fractions and the comparison of fraction size. Their result indicated that manipulatives can enhance the learning of fractions compared to physical manipulatives. Moreover, 60% of the students developed a positive attitude toward the use of manipulatives.

Statement of the Problem

Many children struggle to learn mathematics and often do not achieve success through their learning. This may result from the fact that they do not construct appropriate understanding of fundamental mathematics concepts through their learning strategies. This has left the students with poor performance in both public and internal examinations (Obodo, 2004). Despite the high position offered to mathematics in Nigeria educational system. It is dishearting that approaches, strategies and inappropriate use of manipulative objects for teaching and learning of the subject especially at primary school level are probably not effective in promoting learners activity and providing learner's guided practice enabling them improve their critical, logical and reasoning process to solve problems (Achor, Imoko & Uloko, 2009). Could the use of Van Hiele's model using tangrams improve primary pupil's geometric thinking?

Purpose of the Study

This study aims at investigating primary school pupil's improvement in geometric thinking through Van Hieles's phase –based learning using tangrams. Specifically, the study will determine

i. whether pupil's geometric thinking will improve at the end of Van Hiele's phase of learning using tangrams.

ii. the ability level at first level (visual) and second level (analysis) of geometric thinking skills of high, medium and low ability level of pupil's at the end of Van Hiele's phase of learning using tangrams.

iii. whether male and female pupils will differ in their geometric thinking skills at the end of Van Hiele's phase of learning using tangrams.

Research Questions

The following research questions guided the study;

1. What is the difference between pre-test and post-test mean achievement scores in geometric thinking skills among primary six pupils at the end of Van Hiele's phase of learning using tangrams.
2. What is the difference between pre and post-test mean achievement scores at the first level (visual) of geometric thinking among high, medium and low ability of pupils at the end of Van Hiele's phase of learning using tangrams.
3. What is the difference between pre and post-test mean achievement scores at the second level (analysis) of geometric thinking among high, medium and low ability of pupils at the end of Van Hiele's phase of learning using tangrams.
4. What is the difference between the pre and post-test mean achievement scores of male and female pupils in geometric thinking at the end of van Hieles phase of learning using tangrams.

Research Hypotheses

The following research hypotheses were tested at 0.05 level of significance.

1. There is no significant difference between the pre and post-test mean achievement scores of high, medium and low ability pupils at the first level (visual) of geometric thinking at the end of Van Hiele's phase of learning using tangrams.
2. There is no significant difference between the pre and post-test mean achievement scores of high, medium and low ability pupils at the second level (analysis) of geometric thinking of pupils at the end of Van Hiele's phase of learning using tangrams.
3. There is no significant difference in the post-test mean achievement scores of male and female pupils in geometric thinking at the end of Van Hiele's phase of learning using tangrams.

Methodology

The design of the study was one group pretest- posttest design. This involves pre-testing and subsequent post-testing (Azuka, 2011). The population of the study consists of all primary six (6) pupils in Department of primary education studies demonstration school in Alvan Ikoku federal college of education Owerri Municipal Council Area of Imo State with a population size of 3,462(AIFCE Records unit 2016).The sample of the subjects, consists of 70 pupils from two randomly selected classes from the study area. The sample made up of 40 male and 30 female pupils. The instrument used for data collection was a Geometric Thinking Test (GTT). It was designed based on visualization analysis which is within the capability of primary school learners (Spear 1993). The test consisted of 20 multiple choice items. The construction of the instrument was guided by table of specification. . GTT was used to determine the status of pupils in geometric test. This was used to classify pupils into different scoring/ability groups, i.e, high, medium and low scoring abilities. The maximum score was (100). The classification of students was be done as follows: High scoring students (Upper 25%), Medium scoring students (Middle 50%) Low scoring students (Bottom 25%) The face and content validity of the instrument was determined by two experts in mathematics education and one expert in measurement and evaluation. The reliability of the instrument was determined by administering it on groups of students outside the study groups who had the same characteristics with the study group. The instrument was re-administered on them after two weeks interval using test-retest approach. Correlating the two scores using Pearson product moment, the reliability was found to be 0.87. Before administering the post-test, the groups were pre-tested to ensure level of background knowledge on geometric thinking skills. After that, the same groups were exposed to how to apply Van Hiele's (1986) five phases of learning in geometry. These five (5) phases progress from one level to the next involving inquiry, guided orientation, explication, free-orientation and integration .Activities Prior to Intervention Pupils were individually asked to cut a lined tangram square (17.5x17.5 cm) into 7 pieces. The 7 pieces were numbered on their topsides for reference in directions and discussions of the activities. After students have explored and familiarized them with the tangram pieces, students were taken to **Inquiry Phase:** This phase discovers certain structures by examining holistically examples and non-examples. At this initial stage, students worked cooperatively in a group of 3-4. They were required to manipulate, construct and recognize geometric shapes by using a combination of tangram and concrete objects in their surroundings. This activity leads students to get acquainted with the different geometric shapes. This activity also leads students to notice that joining the tangram pieces sometimes make a shape that is not the same as one of the original pieces. For example two small right triangles will become a square.

Guided Orientation: To examine the properties of the geometric shapes. At this stage, the learners explore the 2-dimensional shapes through carefully guided activities in order to record the properties of the shape. For example, while examining an equilateral triangle, students found that it has such property as three equal sides; three equal angles and three symmetries. **Explication**: This introduces terminology for the properties and different types of polygons. At this stage, teachers introduced new terms for describing the properties and different types of the geometric shape using accurate and appropriate language. For example:- congruent, corners, straight sides, right angles, face, equilateral triangle, square, quadrilateral, regular and irregular polygons, pentagon, hexagon, heptagon and octagon.

Free Orientation: This explores new geometric shapes. The students learnt by doing more complex tasks to find their own way in the network of relations. For example, by knowing properties of pentagon, students investigated these properties for a new shape, such as hexagon, heptagon and octogan.

Integration: This summarizes the properties of a geometric shape. At this stage, students began to build an overview of all that they have learned about a geometric shape. For example, students composed a rule that an octagon has eight equal sides; its corners are the same-all are equal angles; and it can be folded to exhibit 8 line symmetry. The process lasted for three weeks after which a post-test was administered on both groups. The post-test instrument was a rearranged version of the pretest. The data collected were analyzed using mean to answer the research questions while the hypotheses were analyzed using analysis of covariance (ANCOVA) tested at 0.05 level of significance.

Results

Research Question One: What is the difference between pre-test and post-test mean achievement scores in geometric thinking skills among primary six pupils at the end of Van Hiele's phase of learning using tangram.

Table 1: Summary of pupil means achievement scores (Pre and Post –test)

Group	N	Mean	SD	Difference in Mean Gain
Pre-test	70	73.80	3.60	18.33
Post-test	**70**	92.13	3.54	

Result in table 1 shows that pupil's had pre-test mean score of 73.80 and SD 3.6O while post –test mean score of 92.13 and SD 3.54. This implies that geometrical thinking of pupils was improved with a high mean difference of 18.33 after Van Hiele's phase of learning using tangram.

Research Question Two

Table 2: Summary of mean achievement scores in first level (visual) and second level (analysis) of geometric thinking test.

Ability Level :		Visual and	Analysis		
	N	**Test**	**Mean**	**SD**	**Mean Difference**
High	11	Pre-visual	58.37	8.17	15.05
		Post-visual	78.42	12.13	
Medium	33	Pre-visual	51.07	9.31	9.18
		Post-visual	60.25	18.43	
Low	26	Pre-visual	27.34	10.21	
		Post-visual	53.72	15.67	26.38
High	11	Pre-Analysis	69.24	5.63	10.21
		Post- Analysis	79.45	9.87	
Medium	33	Pre-Analysis	64.33	15.31	6.40
		Post- Analysis	70.73	8.69	
Low	26	Pre-Analysis	35.47	16.09	
		Post- Analysis	59.62	12.09	24.15

Result in table 2 indicated different mean scores among the different achievement groups of pupils. Low ability pupils were observed to score high mean score of 26.38 on visual while 24.15 on Analysis. This implies that low ability pupils showed improvement compared to medium and high ability pupils as well as visualization and analysis level of geometric thinking skills.

Research Question Three: What is the difference between the mean achievement score of male and female pupils in geometry thinking at the end of van Hieles phase of learning using tangrams.

Table 3: Mean achievement scores of male and female pupils.(Pre-Post test)

Gender	N	Mean	SD	Difference in Mean
Male(pretest)	40	56.23	5.50	
Female(pretest)	30	57.90	3.72	1.67
				0.72
Male(posttest)	40	78.85	4.39	0.95
Female(posttest)	30	77.90	4.41	

Result in table 3 indicate that the male pupils had a mean achievement gain of (mean 1.67, SD 1.78) while the female pupils had (mean 0.95, SD 0.02). This gave a difference in mean achievement gain of 0.72 in favour of the male pupils.

Hypothesis testing

HO1: There is no significant difference between the mean achievement scores of high, medium and low ability pupils in geometric thinking at the end of Van Hiele's phase of learning using tangrams

Table 4: ANCOVA for post-test score of high, medium and low ability pupils in geometric thinking of Visual and Analysis

Source	Type III sum of squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected model	589.296[a]	3	196.432	10.782	.000	.329
Intercept	1705.368	1	1705.368	93.602	.000	.586
Pre-Test	.268	1	.268	.015	.904	.000
Ability(visual)	**403.090**	**2**	**201.545**	**11.062**	**.000**	**.251**
Ability(analysis)	**413.098**	2	**277.647**	**11.752**	**.000**	**.263**
Gender	12.567	1	12.567	**.529**	**.470**	**.008**
Error	1202.475	66	18.219			
Total	416206.000	70				
Corrected Total	**1791.771**	**69**				

a. R Squared = .329 (Adjusted R Squared = .298)

The analysis in table 4 indicates that there is a significant difference between the mean achievement scores of pupils of different ability levels of geometric thinking. The analysis revealed that there is a significant difference at a value of visual (F = 11.062; Sig = 0.000) .The difference is due high to mean score of the low ability pupils. Therefore, the hypothesis is rejected.

H02: There is no significant difference between the pre and post-test mean achievement scores of high, medium and low ability pupils at the second level (analysis) of geometric thinking of pupils at the end of Van Hiele's phase of learning using tangrams.

The results in table 4 shows ability (Analysis) row 5 F-calculated value (11.752) is greater than P –value (.263). Based on the result the null hypothesis is rejected. This implies that there is a significant difference.

HO3: There is no significant difference between the mean achievement scores of male and female pupils in geometric thinking at the end of Van Hiele's phase of learning using tangrams.

The analysis in table 4 indicates that there is no significant difference in the achievement of male and female pupils in geometric thinking at the end of Van Hieles phase of learning using tangram. The analysis revealed that there is no significant difference as the vale (F = 0.529; Sig. = 0.470). Therefore, the null hypothesis is accepted.

Discussion

The result of this study shows that the use of tangram as an in-class activity following the Van Hiele's five phases of learning has effectively helped students in promoting their geometric thinking. It was also found that Van Hiele's phases of learning using tangrams was able to promote geometric thinking at the van Hiele's level among high, moderate and low ability students. Nevertheless, the result also indicated different degree of effectiveness among the three different achievement groups of students. Low ability students were observed to have greatest improvement score compared to moderate and high ability students in geometric thinking. These results in line with Strom (2009) found that the low achieving children showed more academically successful when using physical manipulative. The results further support the findings of previous researches that tangrams inspire learner's observation and shape analysis and identification (Bohning and Althouse, 1997; Olkun *et al.*, 2005; Yang and Chen, 2010; NCTM, 2003). The results also showed that male and female pupils taught according to the Van Hiele' phases of learning with the help of tangram activities had to explore and discover certain geometric shapes by observation and record directly the properties of the geometric shapes. This implies that gender is not factor.

Conclusion

Incorporating tangram activities in Van Hiele's 5 phases of learning that involves hands-on and investigative approach help students to enhance geometry thinking skills irrespective of gender. Using tangram as manipulative teaching and learning aids allows low achieving students to move easily from visualization to analysis level of geometric thinking.

Recommendation

Based on the result of the study the following recommendations are made:

1. Workshops and seminars should be organized for mathematics teachers to enhance their knowledge on new wave of manipulative materials for teaching mathematics.
2. Mathematics teachers should use manipulative such as tangram in teaching and learning of geometry concepts in primary schools.
3 Mathematics laboratories should be established in primary schools where mathematics teachers and pupils can use for teaching and learning.

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