FORECASTING PERFORMANCE OF LOGISTIC STAR MODEL - AN ALTERNATIVE VERSION TO THE ORIGINAL LSTAR MODELS

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Abstract
This paper proposes an alternative representation of the original version of the Logistic Smooth Transition Auto-Regressive (LSTAR) model. The Logistic Smooth Transition Auto-Regressive (LSTAR) and Exponential Smooth Transition Auto-Regressive (FSTAR) models are frequently used in empirical research. The LSTAR model describes asymmetrical nonlinear adjustment process, while the ESTAR model describes symmetrical nonlinear adjustment process (Sarantis, 1999), in Liews (2002). To Liews et al., the theoretical assumption that exchange rate adjustment is symmetric makes the LSTAR model inappropriate for modelling exchange rate movements, for this reason, the LSTAR model has being neglected in the modelling of exchange rate in the past. They proposed the Absolute Logistic Smooth Transition Auto-Regressive (ALSTAR) model. This version they noted allows a V-shape symmetric adjustment in exchange rate behaviour. In this paper, we propose the Square Logistic Smooth Transition Auto-Regressive (SLSTAR) model which out-performs both the LSTAR and the ALSTAR model in many instances and has the inverted bell-shape of the exponential model which allows a symmetric adjustment in exchange rate behaviour. We have used Monte-Carlo studies and life data to show our claim.

Introduction
The investigation of non-linearities and asymmetries in macroeconomic variables is no doubt a popular area of empirical research. Among various non-linear models reported in the literature are: the exponential auto-regressive (EAR) model, the Markov-switching model (Hamilton, 1989), where changes in regimes are assumed to be governed by the outcomes of an observed Markov chain, the threshold auto-regressive (TAR) model introduced in the time series literature by Howell Tong (see his 1983 and 1990 monographs), here, regimes are defined by the past values of the time series itself, where as in the Markov switching case regimes are defined by exogenous state of the Markov chain. The smooth transition model was first suggested by Chan and Tong (1986), to model a smooth transition between regimes and was subsequently developed by Timo Terasvirta and his various co-authors (see his 1993 monograph with Clive Granger). The former models are characterised by abrupt and sudden change from one regime to another about a particular threshold value. The abrupt regime changes in the threshold model coupled with the difficulties of the non-standard likelihood/least squares functions are unrealistic to many authors (Potter, 1999). This according to Yi-Nung Yang (2002) may not be consistent with the real world observations. Economic variables continue to receive shocks due to decision agents changing behaviour at a given point over time based on many alternative actions facing them. The economy consists of a great number of agents whose behaviour may switch sharply but not simultaneously. The STAR model is a STR model in which the transition variable is an endogenous variable becomes a useful tool. The introduction of smooth transition between regimes allows standard non-linear estimation techniques to be used. The main advantage in favour of STAR models (Nektarios, 2002), is that changes in economic aggregates are influenced by changes in the behaviour of many different agents and it is highly unlikely that all agents react simultaneously to a given economic signals. A STAR model allows that exchange rates alternates smoothly between two regimes. Two STAR models considered by Timo (1994), are the logistic STAR (LSTAR) model, and the exponential STAR (ESTAR) model. Anderson (1992), have successfully applied smooth transition models to a wide range of industrial production series. Yi-N.ung op cit applied STR to model the behaviour of the Dollar/Yen exchange rates. The STAR methodology according to Mark (2002), allows for the possibility that economies do not necessarily jump suddenly from one real exchange rate regime to another, for example between low and high real exchange rates on the basis of a single real exchange rate shock.

Inugunn (2002) applied smooth transition regression model to investigate possible instability
and non-linearity in their model for residential consumption of electricity in Norway. Mark op cil applied STAR models to US dollar real exchange rates of thirteen Latin American countries and found non-linear behaviour in seven countries with the LSTAR model capturing the non-linearities in six. The LSTAR model has been successfully applied by Terasvirta and Anderson (1992) and Terasvirta, Tyssofheim and Granger (1994) to characterise the different dynamics of industrial production indexes in a number of OECD countries during expansions and recessions. The ESTAR model has been applied by Michael, Nobay and Peel (1997) and Taylor, Peel and Sarno (2000) to model real exchange rates. It has also been applied to real effective rates by Sarantis (1999). The issue of a choice between the alternative specifications: LSTAR and ESTAR models in place of a linear model is beginning to generate ripples especially as it relates to the behaviour of the logistic smooth transition autoregressive model. According to Liew (2002), exponential smooth transition autoregressive (ESTAR) model is widely adopted in the exchange rate study as its symmetrical distribution matches that of the symmetrical exchange rate adjustment behaviour. In contrast the logistic smooth transition autoregressive model is discarded by most researchers in priori in their exchange rate modelling exercises due to its undesired property of being asymmetric. They investigated the validity of the claim by researchers that the ESTAR exchange rate model is superior to the LSTAR exchange rate model on the basis of forecast accuracy. They found that this claim is merely theoretical, as they could not find any empirical evidence to support it. They however are of the opinion that LSTAR model should not be dropped just like that and so gave an alternative reparameterised version of the LSTAR model known as the absolute logistic smooth transition (ALSTAR) model.

It is the purpose of this study to also give an alternative re-parameterised version of the original LSTAR model that performs better than the ALSTAR model proposed by Liews et al in some cases. We have used simulation and life data (monthly data on Nigerian Nominal Exchange Rate and the Seasonally Adjusted average Monthly Money Supply billion $ (ml)) for this study.

The Models

A smooth transition autoregressive model of order $p$ as given by Terasvirta (1994) is defined as:

$$y_t = \phi^d x_t + (\phi_1 - \phi_0)y_{t-1}^d F(y_{t-1}^d), \quad \varepsilon_t = \varepsilon_{t-1} \quad t = 1, 2, ..., T$$  \tag{1}

where $y_t$ is a stationary series, $F(.)$ is a continuous transition function which is monotonically increasing, twice differentiable and bounded by 0 and 1, $y_{t-1}^d$ is the transition variable, $x_t = \{y_{t-d}, y_{t-d-1}, ..., y_{t-d-p}\}$, $\phi_i = \{\phi_{i1}, \phi_{i2}, ..., \phi_{ip}\}$, $i = 1, 2, ..., p$, and $d$ is the delay parameter.

The Logistic transition functions in Terasvirta (1994), is:

$$F(y_{t-d}) = (1 + \exp\{-\gamma(y_{t-d} - c)\})^{-1}, \quad \gamma > 0 \tag{2}$$

where $\gamma$ measures the smoothness of transition from one regime to another, $c$ is some threshold value.

Using (2) in (1) give rise to the LSTAR model. He suggests that (2) be re-parameterised as:

$$F(y_{t-d}) = (1 + \exp\{-\gamma(y_{t-d} - c)\})^{-1} - 0.5 \tag{3}$$

The Exponential transition function is defined as:

$$F(y_{t-d}) = (1 - \exp\{-\gamma(y_{t-d} - c)^2\}), \quad \gamma > 0 \tag{4}$$

Using (4) in (1) give rise to the ESTAR model. The transition function is symmetric around $c$ which makes the local dynamics the same for high and low values of $y$, whereas, the mid-range behaviour is different. That is, $F(.) \rightarrow 1$ as $y_{t-d} \rightarrow +\infty$ whereas $F(y_{t-d} = c) = 0$, except for a narrow range of values around $c$. Thus for large values of $y$, it becomes difficult to distinguish an ESTAR model from a linear one.

The Absolute Logistic transition function as proposed by Liew (2002), is:
\[ f(y,\cdot_{-1}) = (1 + \exp\{-y(|y_{-d} - c)|\})^{-0.5}, \quad y > 0 \] (5)

where \(|.| \) means absolute value, other parameters retain their definitions.

Using (5) in (1) gives rise to the absolute logistic STAR (ALSTAR) model.

Our proposed alternative, the Square transition function is;

\[ F(y^*) = (1 + \exp\{-rO>L - c\})^{-1} - 0.5, \quad y > 0 \] (6)

Using (6) in (1) gives rise to the square logistic STAR (SLSTAR) model.

A plot of the transition functions in (3), in (5) and in (6) with respect to the transition variable are shown in Figure I.

![Figure 1: The transition function of LSTAR (left panel), ALSTAR (middle panel) and SLSTAR (right panel) vs the transition variable. Values in the plots are form LSTAR(2) DGP.](image)

From this figure, it is clear that logistic transition function has S shape distribution (left panel), the absolute transition function (middle pane!) has a V shape, although symmetric, it does not look like an inverted-bell shape which is the characteristic of the distribution of the exponential transition function, the square logistic transition function (right panel). The square logistic transition function being proposed has a shape similar to the shape of the exponential transition function. Unlike the sharp vertex of the ALSTAR model, this version allows a better U-shape (similar to the inverted bell-shape of the exponential) symmetric adjustment process of the exchange rate towards the mean of \( y \). We shall show that this version of LSTAR model specification is also capable of describing the

SN mmetrical behaviour in exchange rate behaviour.

The rest of the paper is organised as follows. Section 2 presents the Methodology of our study. In section 3, the specifications, estimation, and evaluation of the model are examined. Section 4 contains the discussion of the results and the last section concludes.

Methodology

(a) We first perform a Monte Carlo experiment by simulating two sets of data from the data generating process (DGP):

\[ y_t = 1.8y_{t-1} - 1.06y_{t-2} + (0.02 - 0.9y_{t-3} + 0.795y_{t-2})F(y_{t-3}) + s_t \]

Terasvirta (1994) with

\[ F(y_{1-d}) = (1 + \exp(-10(y_{1-d} - 0.02)))^{-1} \]

\[ F(y_{1-d} - 0.02))^{-1} \]

\[ d \text{ is the delay parameter } 1 < d < 4. \]

Sample sizes 50, 100, 250, and 500 are used, \( e, \sim iid(0,0.02) \). The experiment is repeated 1000 times and the out-of-sample forecast errors are obtained. The ratio of the number of best forecast (minimum forecast error) to the total trial is recorded in table 3 below.

(b) Two real life data (the Nigerian Nominal Exchange Rate (1974:01-1993:12) and Seasonally Adjusted Average Monthly Money Supply billion $ (1975:01-2004:03) are used for practical purposes. The sample period in both cases are divided into two parts. The first part is used for model estimation while the second part is for assessing the out-of-sample forecast performance of the models. As our estimation with (b), requires stationary series, the standard augmented Dickey Fuller unit root test is performed on the life data. The two series are integrated of order 1. We have used the first differences of the logarithms of the two series which are 1(0) to conduct the study.
Linearity Testing

This aspect concerns the live data used in this study. The initial testing for the presence of non-linearity in the exchange rate series is based on three stages. First, specify a linear AR model for the exchange rate series. The selection of the lag length $p$ is determined. The selection of $p$ is based on the Schwarz Information Criteria and Ljung-Box statistics for auto-correlation. Second, test for the presence of non-linearity in the residual from the chosen AR model. This is achieved by performing the following auxiliary regression:

$$e_t = \beta_0 + \beta_1 x_t + \beta_2 x_{t-1} + \beta_3 x_{t-2} + \beta_4 x_{t-3} + \beta_5 x_{t-4} + \beta_6 x_{t-5} + \xi_t$$

The hypothesis to be tested is $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Equation (7) is estimated for different values of $d$. The $d$ with the least $/7$-value, least AIC and highest $R$ is selected as the optimum delay parameter in the latter estimation. Third, select the appropriate type of smooth transition model (LSTAR or ESTAR for the series). For the third stage, we follow the suggestion of Dijk et al (2000) and run the following sequence of null hypotheses:

$$H_{04}: \beta_4 = 0$$
$$H_{01}: \beta_1 = 0 / \beta_4 = 0$$
$$H_{02}: \beta_2 = 0 / \beta_4 = \beta_3 = 0$$

If $H_{04}$ is rejection, fit LSTAR model. If $H_{01}$ is accepted but $H_{02}$ is rejected, fit ESTAR model. If however, $H_{01}$ and $H_{02}$ are accepted but $H_{04}$ is rejected, fit LSTAR model. A strict application of this procedure according to Granger and Terasvirta (1993), and Terasvirta (1994) can however lead to the wrong conclusion. The suggestion of Sarantis (1999), that the sequence of tests above be performed and the STAR model selected on the basis of the test with the least $/7$-value is followed.

Forecasting Evaluation

The in-sample and out-of-sample performances of the SLSTAR model over the forecast horizon of $n$ over the respective periods are evaluated by taking the LSTAR model as the benchmark. The criterion is the ratio of the forecast error (RMSE) with the forecast of the benchmark model as denominator - Liews op cit. The Meese and Rogoff (1988), MR statistic is used to check the statistical significance of this criterion. The MR is defined as:

$$MR = \frac{\bar{MR}}{\bar{MR}^*} \sim N(0, 1)$$

where $\bar{MR}^* = \frac{\bar{MSE}_1}{\bar{MSE}_0}$ is the MSE of two rival models; with $\bar{MSE}_0 = \sum_{i=1}^{n} u_i^2 / n$, $\bar{MSE}_1 = \sum_{i=1}^{n} v_i^2 / n$, and $\bar{MSE}_0 / \bar{MSE}_1$ is the average ratio of forecast errors of model j and n is the number of forecasts.

For this test, $H_0: MES_1 = MES_0$, (models have the same forecast accuracy). If $MR > Z_{a/2}$ for $n$ is large, reject the null of equal forecast accuracy, otherwise accept the null. If $n$ is small, the student $/a$ statistic is used.

Specification, Estimation, and Evaluation of the model

We first test for unit root on the first difference of the logarithm of the two series and find that they are 1(0) that is, the two series are stationary in their first differences. The next step is to specify the AR($p$). An AR(1) was selected for the exchange rate while AR(6) was selected for the Seasonally Adjusted
average Monthly Money Supply billion $ (ml) The selection is based on the minimum AIC criteria. We perform the auxiliary regression of (7) with the corresponding hypothesis:

\[ H_0 \mathbin{;} (3, =p^* = P_4 = 0 \text{ (model is linear).} \]

Values of \( d \) (delay) between 1 and 5 are used and the regression with the minimum AIC is selected. This null hypothesis was rejected in the two cases. The rejection of the null implies that these series adjust nonlinearly as characterised by the STAR model. Following the step of Sarantis op cit, a sequence of F tests on (8) is conducted. An LSTAR(I) model is identified and fitted for the exchange rate model with \( d = 2 \), Also, an LSTAR (6) model is identified and fitted for the in I series with \( d = 4 \).

**Forecast Performance of the Monte-Carlo Experiment**

Table I shows the rate of out-of-sample performance from 500 trials for each of the three models (ALSTAR, LSTAR and SLSTAR) under the sample sizes: 50, 100, 250, and 500 with two smooth parameters 10 and 100. The samples are such that fifty or more observations at the ends were discarded to remove the initial effects.

<table>
<thead>
<tr>
<th>( d )</th>
<th>( 10 )</th>
<th>( 50 )</th>
<th>( 100 )</th>
<th>( 250 )</th>
<th>( 500 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10 )</td>
<td>0.329</td>
<td>0.340</td>
<td>0.340</td>
<td>0.350</td>
<td>0.497</td>
</tr>
<tr>
<td>( 50 )</td>
<td>0.638</td>
<td>0.647</td>
<td>0.670</td>
<td>0.607</td>
<td>0.497</td>
</tr>
<tr>
<td>( 100 )</td>
<td>0.329</td>
<td>0.340</td>
<td>0.340</td>
<td>0.350</td>
<td>0.497</td>
</tr>
<tr>
<td>( 250 )</td>
<td>0.638</td>
<td>0.647</td>
<td>0.670</td>
<td>0.607</td>
<td>0.497</td>
</tr>
<tr>
<td>( 500 )</td>
<td>0.329</td>
<td>0.340</td>
<td>0.340</td>
<td>0.350</td>
<td>0.497</td>
</tr>
</tbody>
</table>

From Table I, at \( d=1, 2, 3 \) and 4, and alpha equals 10, the SLSTAR performs better than the ALSTAR for all sample sizes; 50, 100, 250, and 500 except at size 500 for \( d = \) \( 1 \) and 4 size 250 for \( d = 2 \), and size 50 for \( d = 3 \) respectively. For alpha equals 100, ALSTAR performs better than the SLSTAR at \( rf=2 \), but the SLSTAR performs better than ALSTAR at \( d = 4 \). This comparison leaves out LSTAR since it is the DGP.

**Estimated Models from Life Data**

The fitted models are tabulated in table 2 in the appendix. The values in brackets are \( \gamma \) values of the corresponding estimates. The null hypothesis of no auto-correlation up to order 20 could not be rejected at 5% level for both series. The null of no auto-regressive conditional heteroskedasticity (ARCH) in the residuals is accepted at 5% level for ER series but rejected for miseries. We conclude that the selected models are adequate for both series.

**Performance of the SLSTAR(p) Model**

Table 3: Performance of the Three Models (Ratio of Rmse) for Compared, Exchange Rate Series (Left Panel), Mj Series (Right Panel)

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( LSTAR(i) )</th>
<th>( ALSTAR(I) )</th>
<th>( SLSTAR(I) )</th>
<th>( LSTAR(6) )</th>
<th>( ALSTAR(6) )</th>
<th>( SLSTAR(6) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( LSTAR(i) )</td>
<td>1.000</td>
<td>1.000</td>
<td>0.9872</td>
<td>1.000</td>
<td>0.9986</td>
<td>0.9356</td>
</tr>
<tr>
<td>( ALSTAR(I) )</td>
<td>1.000</td>
<td>1.000</td>
<td>0.9872</td>
<td>1.000</td>
<td>0.9986</td>
<td>0.9356</td>
</tr>
<tr>
<td>( SLSTAR(I) )</td>
<td>1.000</td>
<td>1.000</td>
<td>0.9872</td>
<td>1.000</td>
<td>0.9986</td>
<td>0.9356</td>
</tr>
</tbody>
</table>

From Table 3, (left panel) the ALSTAR (1) and the LSTAR (1) performed equally. The SLSTAR (1) model however marginally out-performed both the ALSTAR (1) and the LSTAR (1) model. From (right panel), the ALSTAR (6) marginally out-performed the LSTAR (6) model. Whereas the SLSTAR (6) model out-performed the LSTAR (6) and ALSTAR (6) models.
The out-sample performance of the estimated SLSTAR(/>) model over the forecast horizon of \( n = 15 \) observations (1992:10-1993:12), for the exchange rate and \( n = 17 \) observations (2002:11-2004:03), for the m1 series are evaluated using both LSTAR(p) and the ALSTAR(/>) models as benchmarks. The criterion used in these cases, is the ratio of the forecast error (RMSE) of the SLSTAR(p) to that of the LSTAR</> and that of the ALSTAR(/>) models. We then compute the Meese and Rogoff (1988), MR statistic to assess the significance of this criterion. The MR statistics are shown in Table 4 (left panel) for exchange rate and (right panel) for m1 series.

**Table 4: MR for Comparison of Model For Exchange Rate (Left Panel), and M1 Series (Right Panel)**

<table>
<thead>
<tr>
<th></th>
<th>LSTAR(1)</th>
<th>ALSTAR(1)</th>
<th>SLSTAR(1)</th>
<th></th>
<th>LSTAR(6)</th>
<th>ALSTAR(6)</th>
<th>SLSTAR(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTAR(1)</td>
<td>-</td>
<td>0.02897</td>
<td>-0.001205</td>
<td></td>
<td>-</td>
<td>-0.8929</td>
<td>1.5497</td>
</tr>
<tr>
<td>ALSTAR(1)</td>
<td>-</td>
<td>-</td>
<td>-0.001205</td>
<td></td>
<td>-</td>
<td>-</td>
<td>1.4882</td>
</tr>
<tr>
<td>SLSTAR(1)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

From Table 3, with table 4 left panels, the RMSE ratio of the SLSTAR to the LSTAR model for the out-of-sample performance at forecast horizon \( n = 15 \) is 0.9872 (-0.001205). The MR is in the bracket, This indicates a marginal performance of the SLSTAR model over the LSTAR model. The difference in performance is however not statistically significant at 5% level since \( t_{0.025} = 2.145 > 0.001205 \). This is exactly the same result for SLSTAR versus the ALSTAR model. Similarly, Table 3 with table 4 (right panel), the RMSE ratio of the SLSTAR to the LSTAR model for the out-of-sample performance at forecast horizon \( n = 17 \) is 0.9356 (1.5497). This indicates a better performance of the SLSTAR model over the LSTAR model, although the difference in performance is also statistically not significant at 5% level since \( t_{0.025} = 2.120 > 1.5497 \). The RMSE ratio of the SLSTAR to the 'ALSTAR model for the out-of-sample performance at forecast horizon \( n = 17 \) is 0.9370 (1.4882). This also indicates a better performance of the SLSTAR model over the ALSTAR model, although the difference in performance is also statistically not significant at 5% level since \( t_{0.025} = 2.120 > 1.4882 \). These results notwithstanding should indicate a preference for SLSTAR model.

**Discussion of Results**

The results of the Monte-Carlo experiment, indicate that a better model than the LSTAR and ALSTAR models is possible. The SLSTAR show better symmetric representation than the ALSTAR(p) model. It performs generally better with small alpha, and with big alpha, when the delay parameter is greater than 2. The LSTAR tends to perform better when alpha is large. With the life data, linearity is rejected in favour of STAR specification for the two series. LSTAR (1) model with \( d = 2 \) is found to be adequate for the exchange rate of the Nigerian Naira. LSTAR (6) model with \( d = 4 \) is found to be adequate for the Seasonally Adjusted Average Monthly Money Supply. From table 2, the three models consistently estimate the parameters. The appropriate model may just be selected based on the adjusted R-square value and the one with the least out-sample forecast error. The null hypothesis of no auto-correlation up to order 20 was accepted at 5% level, which shows that the errors are white noise. The models adequacy is not in doubt. The performance of the SLSTAR(1) model is better than that of the benchmark models LSTAR (1) and ALSTAR (1) in the case of exchange rate series. The performance of the SLSTAR(6) model, is better than that of the benchmark models LSTAR (6) and ALSTAR (6) in the case of the Seasonally Adjusted Average Monthly Money Supply. These results are in line with the Monte-Carlo experiment. For the exchange rate data, the transition parameter have shown a relatively smoother switch (-4.3 and -53.4) and \( d = 2 \) with a sample size less than 250. For the Seasonally Adjusted Average Monthly Money Supply, the transition parameter of 383.3 and 647.534 show abrupt change between regimes but the parameter is -16.4 for our proposed alternative. This shows a smoother change as against that of ALSTAR and LSTAR models.

**Conclusion**

The application of LSTAR models in the characterisation of the behaviour of economic variables although may seem inappropriate due to its asymmetric nature, the SLSTAR model being
proposed is a good symmetric substitute whenever it is observed that the logistic model is appropriate for modelling a given economic series. The SLSTAR models used in this study are seen to produce better out-of-sample performances than both the ALSTAR and the LSTAR models. This shows that the SLSTAR model is capable of explaining the nonlinear adjustment behaviour in some economic variables.

References


### Appendix

Table 2: **Estimated Parameters From the Life Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LSTAR Model</th>
<th>Absolute LSTAR Model</th>
<th>Square LSTAR Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>_dlo(ER)</td>
<td>0.15427 (0.0877)</td>
<td>-0.15427 (0.0877)</td>
<td>0.14100 (0.00001)</td>
</tr>
<tr>
<td>dlog(ml)</td>
<td>0.001321 (0.0046)</td>
<td>0.000125 (0.0042)</td>
<td>0.0020 (0.0087)</td>
</tr>
<tr>
<td>j1IMERL</td>
<td>0.001231 (0.0046)</td>
<td>0.00125 (0.0042)</td>
<td>0.0020 (0.0087)</td>
</tr>
<tr>
<td>P_n</td>
<td>—</td>
<td>0.1590 (0.0024)</td>
<td>0.1596 (0.0024)</td>
</tr>
<tr>
<td>P_3,4</td>
<td>—</td>
<td>0.1804 (0.0016)</td>
<td>0.1761 (0.0011)</td>
</tr>
<tr>
<td>P_i</td>
<td>—</td>
<td>0.27643 (0.0002)</td>
<td>0.2814 (0.0000)</td>
</tr>
<tr>
<td>P_3,0</td>
<td>—</td>
<td>0.16917 (0.0619)</td>
<td>-0.13147 (0.2117)</td>
</tr>
<tr>
<td>P_2,1</td>
<td>0.06995 (0.2959)</td>
<td>0.5435 (0.0000)</td>
<td>0.6299 (0.3374)</td>
</tr>
<tr>
<td>P*</td>
<td>—</td>
<td>-0.1817 (0.0511)</td>
<td>-0.2091 (0.019)</td>
</tr>
<tr>
<td>y</td>
<td>-4.3068 (1.000)</td>
<td>-383.30 (0.7494)</td>
<td>-647.534 (0.8386)</td>
</tr>
<tr>
<td>c</td>
<td>-0.5850 (0.9999)</td>
<td>-0.00472 (0.0000)</td>
<td>-0.00472 (0.0000)</td>
</tr>
<tr>
<td>se</td>
<td>0.08994</td>
<td>0.00554</td>
<td>0.00556</td>
</tr>
<tr>
<td>P(Q test)</td>
<td>0.999</td>
<td>0.067</td>
<td>0.999</td>
</tr>
<tr>
<td>P(Q test)</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>R²</td>
<td>0.00313</td>
<td>0.2698</td>
<td>0.00313</td>
</tr>
<tr>
<td>RMSE(n)</td>
<td>0.047945</td>
<td>0.00699</td>
<td>0.047945</td>
</tr>
</tbody>
</table>