THE INFLUENCE OF COMPRESSIBILITY ON ECONOMIC PIPE DIAMETER FOR VISCOUS FLOW

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Abstract

Experimental inferences drawn by researchers on fluid flow have indicated the dependence of nature of flow as well as fluid flow parameters in ducts/pipes on the physical properties of fluid. For turbulent flow condition authors have developed procedures for optimization of conduct size and its dependence on fluid compressibility have been reported .In this work, the iterative optimization procedure for laminar flow in pipeline is reviewed .The resulting optimum function was simulated and validated to generate database for the EXCEL package, which was used to evolve quantitative relationship between fluid compressibility and optimum pipe diameter. Results obtained revealed a linear dependence of the form:

Y — Ifl.Uecti + C , a situation similar to what was reported for turbulent flow where fluids with higher compressibility require larger optimum pipe size.

Key Words: Laminar Flow, Compressibility, Economic Pipe Diameter, Optimization.

Introduction

In recent times, economic approach to design of processes as well as engineering systems, machinery and structures have been receiving utmost attention by researchers. The apparent successes recorded through the engineering economy concept had been aided by available optimization techniques, which are based on minimum cost per unit of time or maximum profit per unit of production *(Peters and Timmerhaus, 1968).* The former have been employed by authors *(Alamu ,Adigun, and Durowoju, ,2002; Ojediran 2003),* for optimization of turbulent fluid flow in pipelines. The reported dependence of fluid flow behaviour on inherent physical properties of fluid prompted other researchers, notably *Akintola; 2003, Alamu,,Adekunle,and Odewole, 2003,* to investigate the influence of fluid properties such as compressibility and density on the optimum diameter of pipes for turbulent flow.

In pipe flow problems, pipe sizes are selected based on the design criteria and economic considerations *(Akintola and Alamu, 2002).* The capital cost of a pipe run increases with diameter, whereas the pumping cost decrease with increasing diameter. Selection of optimum pipe diameter for any type of flow; turbulent or laminar, has therefore been seen as a vital economic decision. To achieve this, optimization procedures were proposed by authors *(Peters and Timmerhaus, 1968),* and were subsequently adopted by later researchers to determine economic pipe sizes for fluid flow using computer simulation *(Alamu et al 2002; Ojediran 2002; Akintola 2003).*

The present work presents a review of theories of fluid dynamics and fluid flow cost concept to complement the earlier works of *Akintola (2003)* by extending the investigation of the effect of compressibility of fluid on economic pipe diameter to laminar flow cases, characterized with low Reynold's number *(*N_{RE} < 2000).

Materials and Methods

It has been reported by authors *(Peters and Timmerhaus, 1968)* that for most types of pipe, a plot of the logarithm of the pipe diameter versus the logarithm of the purchase cost per unit length of pipe is essentially a straight line. Hence the piping cost, incorporating capital and maintenance charges, has been expressed as:

$C_{piping} = Xd^n(1+F)K_F$ .... (1)

where: X = cost per unit length of pipe, (₦mm^{-1})

K_F = maintenance, and capital charges expressed as a fraction of initial cost for completely installed pipe

d = pipe diameter, (mm)

F = constant (fittings and installation cost / cost for new pipe)

n = constant (pipe material dependent).

Another author has indicated that the constant n is a function of the current piping cost *(Sinnot, 1993)*. For steel pipes, this constant has been approximated to 1.5 if $d \geq 25.0mm$ and 1.0 if $d < 25.0mm$ *[5]*. Also, for constant F, authors *(Peters and Timmerhaus, 1968; Alamu et al, 2002, Akintola, 2003)* have used the ratio 7:5.

One of the most widely used equations for pipe flow, which satisfied experimental inferences, is the Darcy-Weisbach formula expressed as: *(Theodore and Lionel, 1967; Douglas et al, 1995)*:

$$h_f = \frac{fLV^2}{2gd} \qquad \text{.....................................} \quad (2)$$

where,

f	=	fanning friction factor,	L	=	length of pipeline, (m),
V	=	linear fluid velocity, (ms^{-1})	d	=	diameter of pipe, (m)
g	=	acceleration due to gravity, (ms^{-2}),	h_f	=	frictional head loss, (m)

The friction factor is dependent on the nature of flow. Expressions therefore abound in literatures *(Peters and Timmerhaus, 1968; Kurmi, 1991; Sinnot, 1993)* relating the friction factor with Reynolds number. For laminar or streamline flow, fanning friction factor has been expressed as:

$$f = \frac{16}{N_{RE}} \qquad \text{.................................} (3)$$

Where, N_{RE} = Reynold's number

Equation (3) above is valid for flow conditions, characterized by $(N_{RE} < 2000)$, The Reynold's number can be written in the form:

$$N_{RE} = \frac{Vd\rho}{\mu} \qquad \text{.....................................} (4)$$

where, μ = fluid viscosity, (Nsm^{-2}),

ρ = fluid density, (kgm^{-3})

The linear velocity of the fluid can be expressed as:

$$V = \frac{4Q}{\pi d^2} \qquad \text{.......................................} (5)$$

Experimental inferences of Osborne Reynolds have been reported *(Coulson and Richardson, 1996)* to postulate the dependence of flow pattern in ducts on the physical properties of fluid. Such properties include density, viscosity and compressibility.

The compressibility, γ , of any type of fluid may be obtain from:

$$\gamma = \frac{1}{\rho} \qquad \text{...} (6)$$

Equations (3), (4), (5) and (6) can be combined to obtain the friction factor for streamline flow as:

$$f = \frac{4\pi\gamma\mu d}{Q} \qquad \text{.......................................} (7)$$

Using equations (7), the fanning pressure drop equation *(Sinnot, 1993; Akintola et al, 2003)*, for laminar flow becomes:

$$P_d = 1.3486074\times10^{13} Q\mu d^{-4}\gamma^{1.84} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

For flow through pipes of constant diameter, Sinnot (1993) expressed the annual pumping cost as;

$$C_{pm} = \frac{P_d Q t C_e}{\rho\eta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where; P_d = pressure drop, (kNm^{-2})

Q = fluid flow rate (kgs^{-1}),

C_e = cost of electrical energy, (₦ $kWhr^{-1}$),

t = operational hours per year; (hr. yr^{-1}),

ρ = fluid density, (kgm^{-3})

η = efficiency of motor and pump, (%)

Akintola (2003) and Alamu et al (2003) have used equation (9) above in the economic analysis of turbulent and viscous flow in pipes respectively.

Equations (6), (8) and (9) can be combined to obtain:

$$C_{pm} = 1.3486074\times10^{13} Q^2 \mu C_e t d^{-4}\eta^{-1}\gamma^{2.84} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

The addition of equations (1) and (10) gives the total annual cost of pumping and piping installation for the pipe flow system. Thus:

$$C_T = \frac{1.3486074\times10^{13} Q^2 \mu C_e t\gamma^{2.84}}{\eta d^4} + (1+F)Xd^n K_F \quad \ldots\ldots \quad (11)$$

The pipe diameter d in equation (11) for which the expression has minimum value represents the economic pipe diameter. In this work, the diameter is determined through computer simulation of the foregoing equations using FORTRAN 77 computer codes. With an initial guess value of the internal diameter of pipe, the nature of flow is determined from equation (4) and the total annual cost evaluated using equation (11). This procedure is repeated with increase in the value of the pipe diameter, and the total cost determined in each case until the total cost function passes through a minimum point.

The above problem has been solved by Durowoju and Alamu (2003) who adopted the analytical optimization approach to obtain an expression for the economic pipe diameter (transformed to a function of compressibility) as:

$$d = 557.6851829\left[\frac{Q^2\mu C_e t\gamma^{2.84}}{(1+F)XK_F\eta}\right]^{\frac{1}{5}} \quad \ldots\ldots\ldots\ldots\ldots \quad (12)$$

Software Assessment

Laminar flow case study due to Durowoju and Alamu (2003), (case 1), applicable under ordinary industrial condition (Table I) was used in testing and ascertaining the validity of the developed program. The above technique was adopted for different hypothetical values of fluid compressibility in the laminar flow case studies of Akintola (2003), (case 2) and Durowoju and Alamu, (2003). In each case, the economic pipe diameter was determined. Subsequently, quantitative relationship between fluid compressibility and the economic pipe diameter was sought through the EXCEL package for the cases considered.

Case Studies

<u>Case 1</u> y = 1.11 lXKTnAg"[1], Q = 4.2kgs"', K_F = 7.5%, F = 1.4,
C_c = N30kWhr"', t - 8000hryr"', n = 0.6, u = 1
OmNsm"[2], X = N4.00mm"\ d,, = 28.90mm, 5d = 0.05mm

<u>Case 2</u> y = (9.8039-1I.3636)X10"Vkg"', Q M.Okgs"[1], K,, = 7.5%
F = 1.4, C_e = N30kWlu', t = 8024hryr-', *r*\ = 0.5,
u = 11.5mNsm"[2], X = W.OOmm"[1], d_0 = 29.5mm, 5d = 0.05mm

Results and Discussion

The output of the program for the flow data of case 1 above is presented in Table I. The results gave good agreement with the manual computations of Durowoju and Alamu (2003), as well as calculated values obtained through the analytical approach [equation (12)]. The pipe diameter, corresponding to the least total cost, as evident in Table I and graphically illustrated in Figure 1, is 30.4mm. The same value was obtained by Durowoju and Alamu (2003), while equation (12) gave 30.41mm.

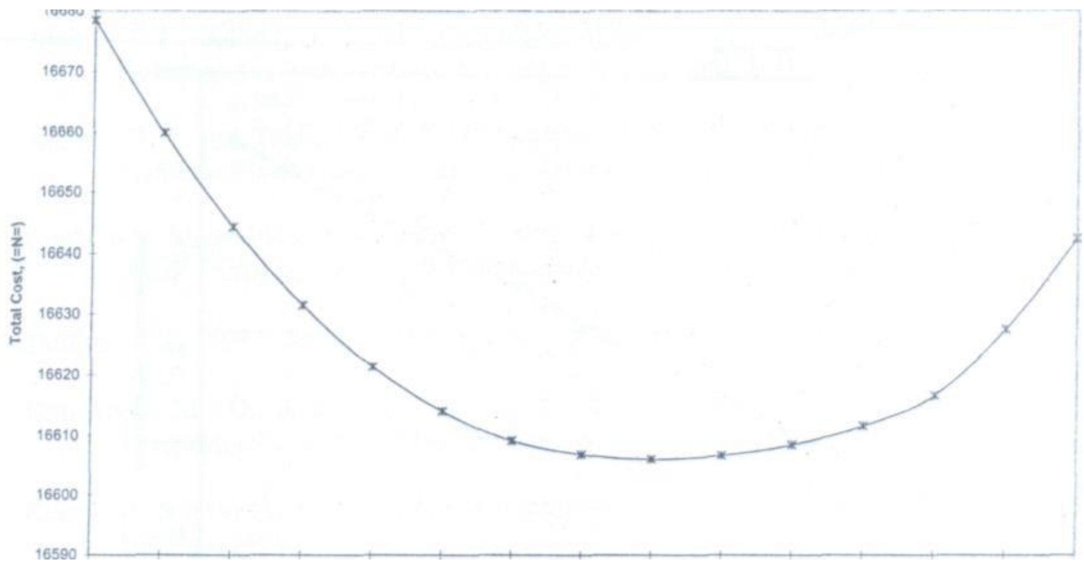
<u>**Table I : Output of the Computer Program for** case **1**</u>

SN. PIPE PRESSURE PUMPING
PIPING TOTAL
DIA DROP COST COST COST

Din.	*uisxjr*	LUii	LUii	LUJI
(mm)	(KN/sq.m)	(=N=)	(=N=)	(=N=)
1 29.30	2.8175	5259.26	11419.15	16678.42
2 29.35	2.7983	5223.52	11448.40	16671.91
3 29.40	2.7793	5188.07	11477.66	16665.74
4 29.45	2.7605	5152.93	11506.96	16659.88
5 29.50	2.7418	5118.08	11536.27	16654.35
6 29.55	2.7233	5083.53	11565.61	16649.14
7 29.60	2.7050	5049.27	11594.98	16644.25
8 29.65	2.6868	5015.29	11624.37	16639.67
9 29.70	2.6687	4981.61	11653.79	16635.40
1029.75	2.65084948.20		11683.23	16631.43
1129.80	2.63314915.08		11712.70	16627.77
1229.85	2.61554882.23		11742.19	16624.41
1329.90	2.59804849.65		11771.70	16621.35
14 29.95	2.5807	4817.35	11801.24	16618.59
15 30.00	2.5636	4785.31	11830.81	16616.12
16 30.05	2.5465	4753.54	11860.40	16613.94
17 30.10	2.5297	4722.04	11890.01	16612.05
18 30.15	2.5129	4690.79	11919.65	

16610.44
19 30.20 2.4963 4659.80 11949.31
16609.11
20 30.25 2.4799 4629.07 11979.00
16608.07
21 30.30 2.4635 4598.59 12008.71
16607.30
22 30.35 2.4473 4568.36 12038.45
16606.81
23 30.40 2.4313 4538.38 12068.21
16606.59
24 30.45 2.4153 4508.65 12098.00
16606.64
25 30.50 2.3995 4479.15 12127.81
16606.96
26 30.55 2.3839 4449.90 12157.64
16607.54
27 30.60 2.3683 4420.89 12187.50
16608.39
28 30.65 2.3529 4392.11 12217.38
16609.50
29 30.70 2.3376 4363.57 12247.29
16610.86
30 30.75 2.3225 4335.26 12277.22
16612.48
31 30.80 2.3074 4307.17 12307.18
16614.35
32 30.85 2.2925 4279.32 12337.16
16616.48
33 30.90 2.2777 4251.69 12367.17
16618.85

34 30.95 2.2630 4224.28 12397.20
16621.48
35 31.00 2.2484 4197.09 12427.25
16624.34
36 31.05 2.2340 4170.12 12457.33
16627.45
37 31.10 2.2197 4143.37 12487.43
16630.80
38 31.15 2.2054 4116.83 12517.56
16634.39
39 31.20 2.1913 4090.50 12547.71
16638.21
40 31.25 2.1774 4064.39 12577.88
16642.27



29.9

29.6

30.6

30.85

30.2 30.4

Pipe Diameter, (mm) FIG.1:

VARIATION OF **ANNUAL TOTAL WITH PIPE** DIAMETER
FOR LAMINAR FLOW IN PIPES

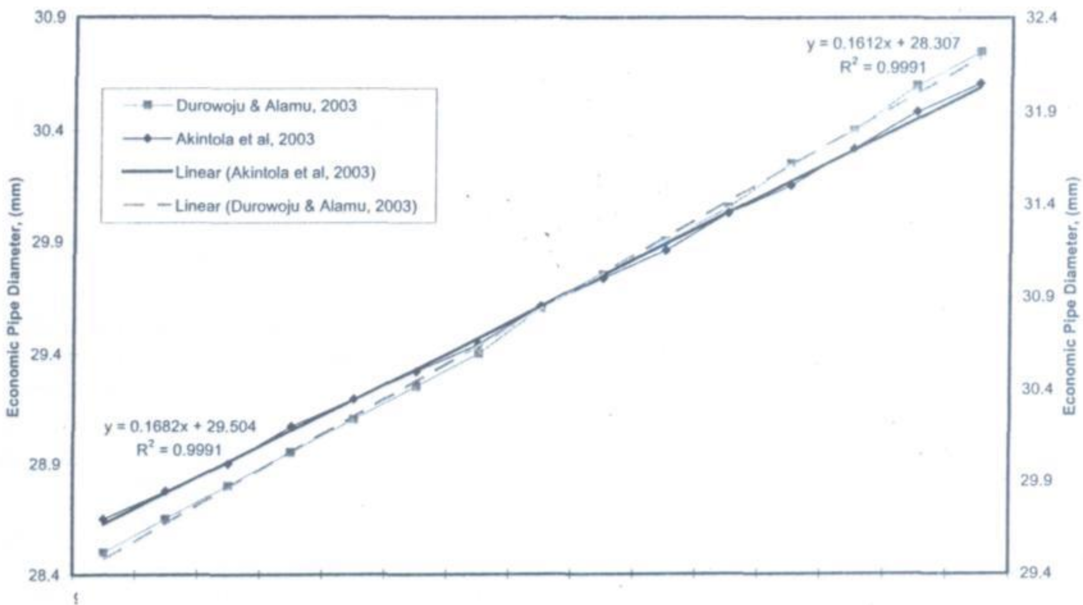
Also, results obtained revealed a decrease in pressure drop with increase in pipe diameter as theoretically implied by equation (8). Pumping cost also decreased while piping cost increases with increase in pipe diameter as earlier observed by Akintola (2003) and Alamu and Taiwo (2003). The above agreements offer some level of justification for the validity of the developed software for base line economic analysis of laminar flow problems in pipelines. However, this does not preclude the judgments of the engineer in respect of other practical considerations which may recommend values other than the theoretically predicted economic values.

Using the two sets of laminar flow cases presented, a total of 30 continuous flow problems were generated, representative of fluids with 15 different values of compressibility [(9.8039-11.3636) X10"^{4}m^{3}kg"] under two distinct flow conditions. Data from these were fed as input into the validated computer program to generate output, similar to Table I; but, summarized as shown in Table II. A graphical illustration of the results is presented in Figure 2.

Table II: Variation of Economic Pipe Diameter with Fluid Compressibility for the cases considered

S/N	**Fluid Compressibility X 10"^{4} (M^3/Kg)**	Economic Pipe Diameter (Case 1)	Case 2)
		(Mm)	
1	9.80392	28.50	29.7
2	9.90099	28.65	29.85
3	10.00000	28.80	30.00
4	10.10101	28.95	30.20
5	10.20408	29.10	30.35
6	10.30927	29.25	30.50
7	10.41666	29.40	30.65
8 ■	10.52631	29.60	30.85
9	10.63829	29.75	30.00
10	10.75268	29.90	31.15

11	10.86956	29.05	31.35
12	10.98900	30.25	31.50
13	11.11111	30.40	31.70
14	11.23595	30.60	31.90
15	11.36363	30.75	32.05



).80392 9.90099 10 10.101 10.204110.3093 10.4167
10.526310.638310.7527 10.8696 10.989 11.1111 11.236 11.3636
Fluid Compressibility, (m3/kg) **FIGURE 2: ECONOMIC PIPE DIAMETER VARIATION WITH FLUID COMPRESSIBILITY**

As evident from Figure 2, the economic pipe diameter increases with fluid compressibility. This presents the same theoretical inference drawn by Akintola (2003), for fluid flow where Reynold's number are higher(N_{RE} > 2000). This suggests that the effect of fluid density on the economic pipe size is independent of the Reynold's number, and hence, nature of flow.

In quantitative terms, as revealed in Figure 2 through the EXCEiL SOFTWARE, the relationship is linear and of the form: $J = HlMecn + C$, where, m and c are real values characteristic of the specific flow. For the cases considered in this work, the respective values are as shown in Figure 2.

Conclusion

Analysis of economic pipe diameter for laminar flow in pipelines was simulated using iterative technique. Results obtained from the validation of the developed software showed that economic pipe diameter increases with fluid compressibility. The optimum pipe diameter was also found to be independent of the Reynold's number.

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Computer Program

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C       PROGRAM: MODIFIED ECONOMIC PIPE DIAMETER FOR LAMINAR FLOW '
C_____________________________________________. _________________________ '
      IMPLICIT REAL*8(A-H,0-Z)
      DIMENSION D(60),Pd(60),Cpu(60),Cpi(60),CTOTAL(60),Re(60)
      OPEN(UNIT=8,FILE='FORUM.OUT)
      WRITE(8,*) Q
      C*************************************** INTERACTIVE DATA INPUT
      PAI=22.0/7.0
      WRITE(*,*)'Input the compressibility of the fluid, (kg/cubic metre)'
      READ(*,*)comp
      WRITE(*,*)'Input the fluid viscosity,(Ns/sq.metre)'
      READ(*,*)FV
      WRITE(*,*)'Supply the fluid flowrate,(kg/s)'
      READ(*,*)Q
     WRITE(*,*)'Input the annual fixed charges including maintenance, 1
     expressed as a fraction of initial cost for completely installed p 2ipe'
      READ(*,*)fK
      WRITE(*,*)'Enter the cost of electrical energy,(=N=/kWh)'
      READ(*,*)eC
      WRITE(*,*)'Input the operation time of the system,(hr/yr)'
      READ(*,*)t
      WRITE(*,*)'Enter the pumping plant efficiency'
      READ(*,*)EP
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 WRlTE(*,*)'Supply the purchase cost per metre lenght of a lmm diam
3eter pipe (=N=)'
 READ(*,*)X
 WRITE(*,*)'enter ratio of total cost for fittings and installation
4to purchase cost for new pipe'
 READ(*,*)F
 WRITE(*,*)'Input a guess value for the pipe diameter'
 READ(*,*)D(1)
 WRITE(*,*)'Supply a small increment in pipe diameter'
 READ(*,*)XD
 WRITE(8,*)'OUTPUT:ECONOMIC PIPE DIAMETER FOR LAMINAR FLOW
 WRITE(8,*)
WRITE(8,*)'
5_'
 WRITE(8,7)
 WRITE(8,9)
 WRITE(8,10)
WRITE(8,*)' 6_' 7
FORMATCAIX/SN.'.IX/PIPE'^X.'PRESSURE'^X/PUMPING'^X'PIPING'
8,5X,'TOTAL')
9  FORMAT(6X,'DIA',4X,'DROP',9X,'COST',7X,'COST',7X,'COST')
10 FORMAT(6X,'(mm)',3X,'(KN/sq.m)',4X,'(=N=)',6X,'(=N=)',6X,*(=N=)')

C ****************************************** COMPUTATION
 DO 30 1=1,40 D(I+1)=D(I)+XD
Re(I)=(4.0*Q)/PAI*FV*D(I)*comp
IF(Re(I) .GT. 2000.0) GO TO 50
Pd(I)=(D(I)**4.0)*(comp**(-1.84))
Pd(I)=1.3486074el3*Q*FV/Pd(I)
Cpu(I)=Pd(I)*Q*eC*t
Cpu(I)=Cpu(I)*comp/EP IF(D(I) .GT.
25.0)THEN Zn=1.5
ELSEIF (D(I) .EQ. 25.0)THEN Zn=1.5 ELSE Zn=1.0 . ENDIF Cpi(I)=X*(D(I)**Zn)
Cpi(I)=Cpi(I)*(1.0+F)*fK CTOTAL(I)=Cpu(I)+Cpi(I) Q
C******************************** writmg Result to File FORUM.OUT
 WRITE(8,20)I,D(I),Pd(I),Cpu(l),Cpi(l),CTOTAL(I)
20FORMAT(1X,I2,2X,F6.2,2X,F7.4,3X,F10.2,3X,F9.2,1X,F9.2)
30 CONTINUE
 WR1TE(8,*)'
8_'
 WRITE(*,*)'Type EDIT FORUM.OUT for the result'
STOP 50 WRITE(8,*)'FLUID FLOW IS NOT
VISCOUS' STOP
 END
```