ON THE CONTROL OF UNIVERSITY GRADE STRUCTURE

S. D. Okonta And E. O. Onobun

Abstract

This work involves the control of university grade structure through promotion control vector. Markov chain models were extensively used for the analysis of manpower planning systems. The movement of staff from one grade to another within the organisation was represented by Markov chain models. Models were constructed and the data collected from personnel department. University of Benin were used to validate the models. An attempt was made to solve the manpower planning problem through the use of the controlled promotion matrix obtained. The maintainable structure through promotion vector yielded a ratio of 1: 0 9 as against I:
4.1 for grade 1 to grade 3.

Introduction

The problem of manpower planning is to determine the number of people by type of gain, staying and loss that result from the effort of an organisation to best meet its further needs in the light of conflicting social - economic organisational objectives and restrictions. Lack of planning for human resources means that organisations are constantly facing crisis because of a shortage of talents or because surpluses exist. Either situation is detrimental to the organization's interest.

In order to improve the operating system the personnel manager needs to employ the knowledge of stochastic processes to predict the future size of human resources desired by the organisation in such a manner that objectives of the organisation, the goals of individual workers and the aims of the society at large are all attained to the highest degree compatible with the workers' situation, some element of forecasting is needed. Lim (1988) "a stochastic process is one in which there is a sequence of chance event involving random variable". This is to say that stochastic models are applicable to any system involving - variability at time passes.

Objectives Of The Study

the aims of this work are to:

(i) Estimate the grade wise distribution of future manpower structure of the academic staff of a University, a case study of University of Benin given that the total number of staff is fixed.

(ii) Find out whether the growth at the top grades is more rapid or likely to continue indefinitely on the long run in relation to the bottom grades.

(iii) Develop a quantitative model, centred on the use of controlled promotion parameter to attain the desired future structure.

(iv) Take a look at the length of time an individuals expects to spend on one grade as well as estimate the probabilities of individual (having entered at various grades) reaching the top grade.

Methodology

Data Collection: - Data were collected from both the primary and secondary sources. The data needed for this work were mainly on the academic staff of University of Benin. On the primary source, structured interview questions were administered to the respondents for completion. Data collected through the secondary sources were gotten mainly from university records such as files, journals, and other relevant literature.

Sampling Procedure:

The research population comprises of all the Nigerian universities. For the purpose of time and cost, the researcher used the purposive sampling technique to select university of Benin as a Federal University that can well represent both the Federal and State Universities. This involves only the personnel department, as it is the only department with relevant information.

Data Analysis:

Quantitative methods of'analysis were employed mainly in the prediction of the university future structure given the present stock. Stochastic process is part of the great game of scientific prediction. Markov chain models Vajda (1978) and Edwards (1983) were used as stochastic models for data analysis.

Data Analysis

THE THEORY OF MARKOV PROCESS

A Markov process is a stochastic system for which the occurrence of a future state depends on the immediately preceding state and only on it.

The probability

Pij = O {Stn = J/St.n = i} is known as one - step transition probability of going from state I to at t_n-l to state j at t„. The transition probabilities from E_f to state Ejj. can be represented in a matrix form.

$$\text{Note: - } \sum_{J=i}^{n} P_{ij} = 1 \text{ for all i}$$

$$P_{ij} \geq O, \text{ for all I and j}$$

Stocks And Fohvs Models

Bartholomew (1973) and Vajda (1978). The Markov Model for manpower planning system ;nn be described by the help of the following notations.

Let t = 0, 1,2, ---- , T planning periods, usually a year.

Let ij = 1, 2, 3, "states" of the system:

Representing the various grades of members of staff of the university.

Let Ni (t) be the number of staff in grade i at the beginning of period t

Let Rj (t) be the number of new lecturers recruited to grade j at the beginning of period t

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Let W i(t) be wastage factors representing the proportion of members of staff in grade i, leaving the system during period t due to retirement, death, resignation e.t.c.

Under certain Markovian assumptions, the stocks in grade j at time t + 1 thus consists of the survivors from t plus the new entrants. The stock at the end of a period can be written as.

$$Nj\ (t + 1) = \sum_{i-i}^{3} X\ ij\ (t)\ Ni\ (t) + Rj\ (t) \text{ for every } j = 1, 2, 3 \ldots\ldots\ldots\ldots (1)$$

Where

Pij (t) = N_{ij} (t) / N_i (t) for every {ij = 1, 2, 3 }

And Njj (t) is the total number of staff promoted from grade i to j.

(

$$\text{Also } R_j = \{R_j (t + I)\} / \sum_{j=i}^{3} R_j (t + I)$$

$$\sum^{3} R_j (t + 1) = \sum_{i=j}^{3} w_i (t) N_i (t)$$

$$\sum_{j\ i}^{3} R_j = 1$$

$\sum^{3} R_j (t + I)$ is the total number of staff recruited and it is directly proportional to the $j = \quad j=i$

number wasted to maintain the desired fixed size.

Wastage probability can be determined

as $W_i (t) / N_j (t)$ for every $I = 1, 2, 3$.

$W_j (t)$ are those that left the system.

The expected number of staff promoted form i to j is

$$E (NP) = 2 \sum^{3} N_i (t) P_{ij} (t) j\ 1,2,3.$$

While the expectation of those recruited to j is

$$E(Nr) = r_j \sum^{3} W_i (t) N_i (t) \text{ for every } j = 1,2,3.$$

Conclusively, the expected stock in grade j is

$$E (N, (t +)) = r_j \sum^{3} N_j (t) P_{ij} (t) + r_j \sum_{H}^{3} W, (t) N_i (t)$$

Simplifying (2)

$$E (N_j (t +)) = \sum N, (t) [P + W^i r] =$$

$$= N (t) H \text{ Where}$$

$$H = P + W'r.$$

Equation (3) can be used to estimate the future stock of the system from year to year using previous year's stock. H. in equation 3 is a stochastic matrix with non — negative elements. It shows how transition from one grade to other was done. The row sum of this matrix must be equal to a unit.

Limiting Distribution Of The System

The interest here is to investigate the state of the structure in the long run, that is, as time t goes to infinity.

The limiting behaviour of $E (N (t))$ as $t \to \infty$ is

$E(N(t)) = ZH'$

By theory of Markov chains,

Lim $II^1 = Q D^1 Q^1 = h$................................4.3.1 $t \to \infty$

Where D is a diagonal matrix with its elements as the characteristic roots of the matrix **P** and the columns of Q are the characteristic vector. Corresponding to the characteristic roots.

$E\{N_{(\infty)}\} = h$......................4. 3. 2.

Z is the total (fixed) size of the system.

Control By **Promotion Vector**

For a fixed size organisation, the aim is finding a super diagonal matrix **P** having non-negative elements with i^{th} row summing to

I - Wi (i = 1,2,3.) satisfying

$N = NP\ \boldsymbol{NW}\ r$

Rearranging equation 4.4.1

$N = NW^{;}\ r$

Equation 4. 4. 2 can be generally written as $N_k\_i P_{k\_1k} + N_k P_{kk} = N_k - {}^r{}_k \sum_{i-1}^{k} Ni\ Wi$

Pij, = 0 expect P_{ij} and $P_{j,j} + |$

Set Pii = I - Wi – $P_{i,i} + {}_I$, --------4.4.4

Those wasted at i, and those promoted to i+1 from i

If k = 1 in 4.4.3

$$N| P, | = N, -r, \sum_{i=I}^{k} Ni\ Wi$$

$$N, (I-Wi-Pn) = N, -r, Z_{i=I}^{k} Ni\ Wi$$

$$N_2 P_{23} = N, P,2 - N_2 W_2 - N_2 w_2 + r_2 1_{i=i}^{k} Ni\ Wi$$

$$P_{23} = (r, + r_2) \sum_{i=I}^{k} Ni\ Wi / N_2 - \sum_{i=I}^{3} Nj\ W, / N_2$$

The above can be written in general forms as i k i

$$Pi, j + I = X r, \{ \sum Ni\ Wi / N_2\} - S\ Nj\ Wj\ /.Ni$$

i = I i = I I i = I

i

Σ Nj Wj / Nj = O
i - I

k

Hence P; , + , = Z r, NjWj/Nj (i=1,2 .... k)
- I 4 |
where

0 < Pi ,j i+ j < I - Wi f= 1,2 .... k

Pi i i means the proportions requiring to be promoted from grade i must be equal to the number leaving from i + I to k divided by the size of grade i.

Steady State Control

let N* = N*P + N* W'r

Represent the maintainable structure.

Since the total size of the system is fixed, it will be convenient to work in terms of the proportion, that is X, = N* / Z where Z is total number of people in the system in order to determine the maintainable set.

Equation 1 can be. re written as.
X(I-P) = X W'r
X = X W'r(1-P)■' ------------ 4.5.2
Post — multiplying both side of (4.5.2) by a columns vectors of I's denoted by P
1= X W'r (I - P)-1
X = (W'r d)-1 ---------------------- 4.5.3
Substitute (4.5.3) into (4. 5. 2)
X = (rd)-1 r (I-P) -1

3 3

X = Σ {r, e (1-p) '} / Σ x\ dj
= i ' i i=I

Where e i is a vector with I in the position and zero therefore

3 3

X = Σ {{ 1-i dj / Σ r, dj }di (ei (1-P)-1 }
=1 (h >.
3

Let ai = ri di / Z { i'j dj
j = i

then X = aj di '1 {ei (1-P) 1 }

Average Length Of Time

Our focus here is to determine the average length of time an individual who enters the system through grade i expects to spend in grade j Expectation of Xy which is given by

$$E(X_y) = \sum_{j=1}^{3} E(X_{jj}) = d_i,$$

Where did is the row sum of (I - P) '

CHANCE TO REACH THE TOP - GRADE

The matrix (I - P) 'can be used to calculate the chance that an entrant at the bottom grade will reach the top grade.

Let qjj denote the probabilities that an entrant to grade spends sometime in grade j before leaving.

If11jj is the (j,j)[th] elements of (I - P)"'

Then 11,, =q,j + (1 — q;j) X

I hetetoie qij — Iljj / 11 j j (i,j — 1. 2, j)....................4. 6.1.D. J Bartholomew 1973**).**

To obtain qy divide the elements in each column of (I - P)"' by the diagonal elements of that column. The diagonal elements must be equal to a unit.

Model Testing And Prediction

we shall validate our formulated models with the following data collected from personnel dept University of Benin.

Academic Staff Data - (1999/2000)

GRADE TYPE	Number in the grade	Number promoted	Number wasted	Number recruited
Graduate assistant	52		2	18
Assistant lecturer	100	5	3	19
Lecturer 1 1	106	31	3	13
Lecturer 1	152	28	7	8
Senior lecturer	203	23	8	15
Associate Prof.	43	39	1	2
Full Prof.	61	20	4	3

For ease analysis, the six grades have be reduced to three in the following manner. Graduate Assistant, Assistant Lecturer and Lecturer 11 as Grade I; Lecturer 1 and Senior Lecturer as Grade I 1 and Associate Professor and full professors as grade 3.

N(0) = (258,355,104)

P = (36, 51, 5)

Wi = (81, 15, 5) =(0.031,0,0.0423,0.0481) r =

(50,24,5) = (0.6329, 0.3038, 0.0633)

$$W'r = \begin{pmatrix} 0.0196 & 0.0094 & 0.0020 \\ 0.0268 & 0.0129 & 0.0027 \\ 0.0304 & 0.0146 & 0.0030 \end{pmatrix}$$

$$P = \begin{pmatrix} 0.8295 & 0.1395 & 0 \\ 0 & 0.8139 & 0.1437 \\ 0 & 0 & 0.9520 \end{pmatrix}$$

p is a super - diagonal matrix which indicate no double promotion and demotion in practice.

$$H = \begin{pmatrix} 0.8491 & 0.1489 & 0.0020 \\ 0.0268 & 0.8268 & 0.1464 \\ 0.0304 & 0.0146 & 0.9550 \end{pmatrix}$$

Expectation of future structure of the university grade is a follows:

E { N(t+) = N (t) H

H { N(I)) = N (( )) H = (232,333,152)

E (N) = (158,242,317).

[lie above results show a steady deteriorating position as the systems becomes increasingly top- heavy. A fair conclusion that can be made here, is that, the recruitment and promotion policies adopted by the university are incompatible with maintaining a structure like N(O)

Limiting Distribution Of The System.

The eigen values obtained from H are 1. 07346, 0.6578317, 0.899085 and the corresponding eigen-vector are.

	{ 0.5577	**-0.4656**	**-0.6872 }**
Q =	**{ 0.5175**	**0.8423**	**-0-1507 }**
	{ 0.6490	**-0.2716**	**0.7107 }**

recall equations 4. 3. 1. & 4. 3. 2.

h = {0.31067424, 0.267815762, 0.421197417 }
E {N (oo)] -717 h - (223, 192, 302}.
The limiting distribution shows an increase at the top grade at a decreasing rate.

Computation Of The Controlled Promotion Matrix
Using equations (4. 4. 4.) & (4. 4. 6)

3

P12 = I **Nj Wj** / Nj=0.07759 J=2

P_{23} = 0.0140991267

Pu = 1 - W,-pl2 = 0.891407365

	(0.891407365	0.077592635	0 }
P=	{ 0	0.943608733	0.014091267}
	(0	O	0.9519 }

Steady State Control
Determination of the maintainable region with the existing promotion vector Recall N* = N*H ; X = N* / Z and X = $a_i d_i^1$.(e (I-P)"[1])

(5.865103845	4.39646193	13.73417184}
(0	5.37345626	16.78621002}
(0	0	21.73911349}

	A{ 0.244422713	0.18321853	0.572358756}
$E(1-P)^{-1}d^{-1}$ =	B{ 0	5.373456236	16.78621002}
	C (0	0	21.73911349}

The maintainable region is the one with vertices A, B, C the coefficients a; (i> 1,2.3)

are: a, = 0. 651935674, a_2 = 0. 288924178, a_3 = 0.059071967.

X = (0159347886, 0.1899523929, 0.651128242)
N* = (115. 136, 466)

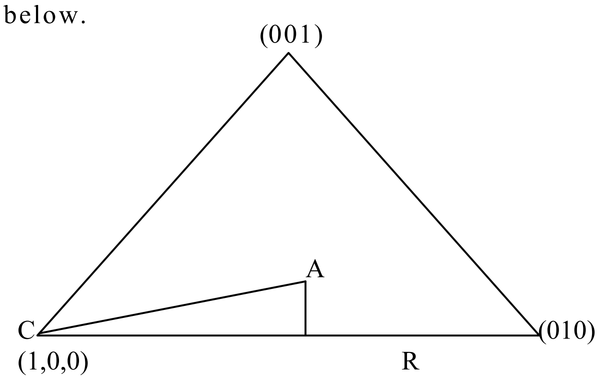
Now. consider the maintainable set with the controlled promotion matrix
Using $*e(I- P^{-i}d_i^{-1}$, we have,

A	=	(0.369832666	0.495118666 0.145048667)	
B	=	(0.	0.77342056 0.226579435)	
C	=	(0	0 1)	

a,	=	0.66181543	a_2	=	0.284444732	a_3	0.05379835
X	=	(0.2381428			0.5476725760		.21418461)
X*	=	(171,393,153).					

This maintainable structure can be represented in a 3 - dimensional Euclidean

Spaces as shown below.



A(0.3598, 0.4951,0.1450); B(0.22657 0.7734 0) C(100).
Any point inside the region indicated by triangle ABC is maintainable.

CHANClvS OF REACHING THE TOP - GRADE

Using equation (4.6.1.) q_{ii}can be obtained as follows

	(1	081818	0.63177)
q„ =	(0	1	0.77217)
	(0	0	1)

from matrix it can be seen that an individual or grade 1 has 81. 82% of reaching the top - grade.

Conclusion

The results from the tested models show that the size of the senior staff will continue to grow in relation to the junior staff. Though the limiting distribution indicated a slight deviation from what was expected. This deplorable situation was, however, arrested through the use of controlled promotion vector.

In the base year of the system, the initial stocks at the different grades indicated a ratio of 2.4:1 for grade 1 to grade 3. at t = 6 tj was 1:2, though the limiting distribution reflected a ratio of 1:
1.6 i.e. (rise, fall, rise,....) not steady. This unsteady state needed to be controlled to reduce financial burden borne by the organisation at a time of budgetary standstill. Using the controlled promotion matrix, the maintainable set was found to be (171, 393, 153) as against the original (1 15, 136, 466). It is therefore advisable for the university management to pursue attainability of this structure (171, 393, 153) with ratio of 1: 0.9 as against 1:4.1.

T he average length of time spend by a staff on a particular grade was studied. Before the control, it was discovered that an academic staff promoted to grade 2 spent at least 5 years there and 16 years on grade 3. After the control, vve noted tht the staff member needs to spend more than 5 years on grade 2 and lesser years on the third grade. The latter is necessary to save cost.

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