INVESTIGATING STUDENTS’ PERCEPTIONS ABOUT WHAT MAKES A GOOD ESTIMATE IN MATHEMATICAL COMPUTATIONS

By

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Abstract
The study investigated students’ perceptions about estimation. The main purpose of the study was to find out students’ thinking about what makes a good estimate in computations. Seventy students were randomly paired, and they studied hypothetical students’ worked examples, and answered questions prompting them to choose which method was easier or which method led to getting an estimate closer to the exact answer. The students’ interactions were videotaped. The study revealed that students’ perceptions about estimation were diverse. The study also revealed that students talked about simplicity and proximity of estimation in different ways. It was recommended among others that teachers should introduce students to multiple estimation strategies and help them acquire experience with each of them.

Many Mathematics educators have maintained that a good number of students find difficulty doing simple computations mentally or estimating the answers to problems (e.g. Case & Sowder, 1990; Reys, Bestgen, Rybolt &Wyatt, 1990). According to Linquist (1989) and Linquist, Carpenter, Silver and Matthews (1983) less than half of the 13-year-olds tested could mentally compute 60*70, and that only 38% thought that 945*1000 should be solved mentally. This inability to do simple calculations in the head or to estimate is a fundamental difficulty to using Mathematics in everyday life. Ordinarily, students should be able to make quick calculations or judgments of numerical magnitude without the help of calculator or paper and pencil.

The ability to quickly and accurately do mental calculations and/or estimations has two benefits, in addition to being a fundamental and real-world skill. These two benefits include (1) allowing the student to check the reasonableness of their answers found through other means, and (2) enabling students develop a better understanding of place value, Mathematical operations and general number concepts (Star, Glasser &
Rittle-Johnson, 2007). Star, Glasser and Rittle-Johnson (2007) pointed out that these benefits are expressed in the recent ‘Adding it up’ report from the National Research Council: “The curriculum should provide opportunities for students to develop and use techniques for mental arithmetic and estimation as a means of promoting deeper number sense”.

The question is: Why is estimation difficult for students? A probable reason might be that the students do not know what makes a good estimate or, it might be because it is not a straightforward issue. The literature on estimation has pointed out two criteria that can be used to adjudge an estimate as good (Star, Glasser & Rittle-Johnson, 2007).

Firstly, an estimate is good if the process of getting the estimate is simple. Simplicity refers to how easy it is to compute an estimate. For example, to calculate an estimate for 32*37, both numbers could be rounded to the nearest ten, and so multiply 30*40. An alternative method would be to round only one of the numbers to the nearest ten, i.e. multiply 30*37. At the elementary level, the first method seems reasonable and likely to be simpler.

Secondly, the other criterion for adjudging an estimate as good is the proximity of the estimate to the exact value. Proximity refers to how close the estimate is to the exact value. For example, to calculate an estimate for 21*37, the 21 can be rounded to 20 and multiply 20*37, or the 37 could be rounded to 40 and multiply 21*40. The former differs from the exact value by 1*37, while the latter differs from the exact value by 21*3. So the former method gives an estimate that is closer to the exact value than the latter method. Thus the former estimate is a better estimate, because of its proximity or closeness to the exact value.

However, the demand for an estimation task may suggest which criterion is paramount. For instance, some situations demand just a very rough estimate, and this calls for simplicity. In such situations simplicity is paramount, even if the estimate is far away from the exact value. On the other hand, some situations call for proximity, even if the computation is far from simple to perform.

In the foregoing examples, it was seen or noted that rounding only one number to the nearest ten always produce more proximal estimations than rounding both numbers. It should be noted that some strategies may present a false illusion of consistently producing more proximal estimates than others. For example, consider which strategy is more proximal for 19*21 and 17*19. It would be noticed that the more proximal of these two strategies depends on the problem.
Therefore the question of what makes an estimate good is not quite a straight
forward issue. The purpose of this study was to investigate how students think about
estimate, in particular what makes an estimate good. This will provide information in
order to understand the difficulties students have with estimations.

Significance of the Study

It is important to investigate how students think about what makes an estimate
good as students’ perception of good estimates might be different from experts’.
According to Star and Madnani (2004) students’ perceptions of what makes a strategy
(for solving linear equation) good were frequently unusual and often diverged from
experts’ views. Having information about what students think about what makes
estimate good would help educators and curriculum developers to incorporate this in to
more effective methods and materials for teaching Mathematics. The language and
reasoning used by students in discussing good estimates would provide details regarding
how students perceive the different strategies for computational estimation.

Methodology

Participants

Seventy Secondary School I students participated in the study. This population
consisted of 38 and 32 students from two classes of a private school in Bauchi
metropolis. There were 37 boys and 33 girls with an average age of 14.7 years. Their
teachers reported that the students were familiar with computational estimation. The
students were randomly paired, and they studied worked examples during a two day
intervention.

Instrument

The instrument for the study was a 14-item computational estimation problem.
The students learned about computational estimation strategies by studying a series of
hypothetical students’ worked examples and by answering questions about the strategies
in the worked examples. Each problem was solved in two different ways and presented
to pairs of students to study and answer the question about the strategy. The question
required the pair of students to answer which strategy was easier or which strategy led
to getting an answer closer to the exact value. Samples of the hypothetical students’
worked examples are shown in Figure 1.
A. Mark and John were asked to estimate 9*24.

Mark’s way:
I rounded both numbers and multiplied 10*20.

John’s way:
I rounded 9 up to ten and multiplied 10*24.

Question: Whose method is easier?

B. Levi and Stephen were estimating 21*39.

Levi’s way:
I rounded both numbers and multiplied 20*40.

Stephen’s way:
I covered the unit’s digits 2 3 and multiplied 2 * 3. I added two zeros.

Question: Whose estimate is closer to the exact value?

Figure 1: Samples of Hypothetical Students’ Worked Examples

Procedure

The students were randomly paired in their intact mathematics classes and the hypothetical students’ work examples were presented to them in their pairs. They discussed the worked examples and answered the questions prompting them to say which strategy was easier or which strategy led to getting an answer closer to the exact value. This intervention was carried out during their regular mathematics lessons. The students’ partner interactions were videotaped and the researcher also took written notes regarding the student interactions with pair groups studying the worked examples, writing their answers to the question and explaining their reasons for the choices of strategies they made. Transcripts of the partner interactions were analyzed with particular attention to the questions relating to students’ perception of the goodness of estimates in terms of simplicity and proximity.

Results

As noted in the literature, the two criteria for adjudging an estimate as good are simplicity and proximity. The transcripts of the partner interactions and the researcher’s notes were analyzed. From the information obtained, it could be said that there were diverse views and reasoning expressed by the students with regard to their perceptions of what makes an estimate good. However, careful analysis of the interactions showed that students perceived simplicity of estimation in three different ways.
Firstly, some students said that simplicity is dependent on the problem type itself. They said there are problems you can estimate easily and there are those you have to write down on paper before you can estimate. They pointed out that estimation with smaller numbers is easier than with larger numbers. For instance, in estimating 172*234, one student said, ‘these numbers are large; it is not easy to do it without writing the numbers on a paper’.

Secondly, some students said that simplicity of estimation was determined by the estimation strategy to be used. A student said the covering of some digits and adding certain numbers of zeros to the result of the multiplication was confusing. Some noted that it was easier to round both and multiply than rounding only one.

Thirdly, some students perceived simplicity of estimation as the one that does not take time to do. For instance, when one pair of students talked about the advantages of two estimation methods, one of them said that the other student’s way was more difficult because, it was going to consume much time.

Proximity
The transcripts analyzed showed that students discussed proximity of estimates to the exact values in two ways. Firstly, some students said that proximity would be realized if only one operand was rounded. Some said that proximity depended on which operand is rounded, and also whether it was rounded up or down, and how close the rounded numbers were to the original. This reasoning is similar to the thinking of the participants in the study by Star, Glasser and Rittle-Johnson (2007).

Secondly, the other way some students talked about closeness to the exact value was how the rounded numbers had effect on the estimate in determining how far away estimates were from the exact values. For example, in determining whose estimate was closer to the exact value of her problem, in estimating 11*78 and 11*28. To estimate 11*78, Mandi multiplied 10*78, and to estimate 11*28, Sally multiplied 10*28. One student said that Sally’s estimate was closer to the exact value. The student explained that 10*78 is 78 less than 11*78, while 10*28 is 28 less than 11*28, and since 78 is greater than 28 then Sally’s was closer.

Discussion
From the analysis of the students’ perceptions of what makes a good estimate with regard to simplicity and proximity, it was revealed that students had diverse perceptions. The analysis revealed that students recognised three characteristics of simplicity of estimation and two of proximity. The three characteristics identified were the problem type, the estimation strategy employed and the time limit for getting the estimate. These three ways students perceived the simplicity of estimation strategies are
similar to the ones identified by Star, Glasser and Rittle-Johnson (2007) in their study with fifth graders. In their study, they identified four characteristics of simplicity of estimation strategies students talked about.

For proximity, some students pointed out that it depended on the strategy to be used; that is how the operands were altered. Some discussed proximity in terms of how far the estimates were from the exact value by noting how the altered operands had effect on the estimates. The reasoning and views expressed by the students were diverse.

**Conclusion and Implication for Education**

The information implies that students’ perceptions about simplicity and proximity of estimation are diverse. This means that one student’s perception of whether an estimation strategy is simple may not be the same with another student’s perception. The diversity in thinking and reasoning implies that two or more students might choose to use different strategies for solving the same estimation problem based on their methods of evaluation of whether a strategy is simple or whether it leads to getting an estimate closer to the exact value. Therefore, teachers should not assume that all students perceive simplicity and proximity of estimation the same. Teachers should introduce students to multiple estimation strategies and help them to acquire experience with each of them. Teachers can also introduce different kinds of statements other students have made regarding estimation strategies, simplicity, and proximity and encourage their students to find out which methods they find to be good and explain their reasoning. Mathematics educators may use such information to incorporate students’ perceptions and reasoning into the content and instruction of mathematics. Similarly, by listening to the diversity of students’ views, teachers can better address specific approaches used by each student.

**Reference**


