PAIRS OF APPROXIMATE FUNCTIONS FROM COLLAPSING BOUNDARIES OF A POSITIVE SERIES

By

Andrew A. Dosomah
Department of General Studies (Mathematics section)
Edo State Polytechnic, Usen

Celestina O. Audu
Department of General Studies (Mathematics section)
Edo State Polytechnic, Usen

And

Sabrena O. Edosomwan
Department of General Studies (Mathematics section)
Edo State Polytechnic, Usen

Abstract

Equivalent numbers have equal decimal expansion such that digits in corresponding positions are equal. Can one construct pairs of approximate functions of limit zero as (n) tends to infinity such that: (1) from 1000 to one billion in steps of multiplier 10000 at least the first two digits from the first non-zero digit to the right of decimal point of the first function are respectively equal to the digits in corresponding positions of the second function. (2) the number of equal consecutive corresponding digits in the first set of consecutive corresponding digits to the right of decimal point in both functions is non-decreasing with increase in (n) in the given range. (3) at n equal one billion, the functions are equal to at least 14 decimal places. Using collapsing boundaries on K_{n+1} *equivalent of sub=intervals within boundaries of positivity conditions of a series associated with unit points of* $1-(n+1)-K_{n+1}$ *exponential integral, the paper shows how to construct approximate functions that satisfy the conditions stated above and gave some simulation result on value characteristics of such functions.*

***Keywords**:* Approximate functions, collapsing boundaries, consecutives digits, unit point, convergent series.

Equal numbers have equal consecutive digits in corresponding positions of their decimal expansion. For example,

$\frac{3}{7} = \frac{6}{14}$ are equivalent fractions,

$\frac{3}{7}$ = 0.428571428…

$\frac{6}{14}$ = 0.428571428…

Can one construct pairs of approximate functions that fulfill the stated conditions in the abstract of this paper?

Two challenges for the equality of corresponding digits of the decimal expansion of approximate functions are:

1. Although two approximate functions may have the same limit, their rate of approach to the limit may be different.
2. Mere proximity may not be a guarantee for the existence of such functions. Consider the pair of approximate functions $\frac{1}{n}$ and $\frac{1}{n+0.1}$, these two functions have limit zero as (n) tends to infinity.

Table (1.1a) shows the values of $\frac{1}{n}$ and $\frac{1}{n+0.1}$ at n = 10000 and n = 100000.

Function	n = 10000	n=100000
$\frac{1}{n}$	0.0001	0.00001
$\frac{1}{n+0.1}$	0.00009999900001	0.00000999999

Table (1.1a)

We observe that $\frac{1}{n}$ and $\frac{1}{n+0.1}$ are very close, the first two digits from the first non-zero digit of $\frac{1}{n}$ at n = 10000 is (10) in positions 4 and 5 to the right of decimal point. The first two digits from the first non-zero digit of $\frac{1}{n+0.1}$ is (99) in positions 5 and 6.

The first two digits from the first non-zero digit of $\frac{1}{n}$ at n =100000 is (10) in positions 5 and 6, the first two digits from the first non-zero digit of $\frac{1}{n+0.1}$ is (99) in positions 6 and 7. The first two digits from the first non-zero digit of $\frac{1}{n}$ and $\frac{1}{n+0.1}$ are neither equal nor in corresponding position when n = 10000 and when n = 100000.

The number of equal consecutive corresponding digits in the first set of equal consecutive corresponding digits to the right of decimal point of $\frac{1}{n}$ and $\frac{1}{n+0.1}$ is (3) at n = 10000 and (4) at n = 100000. Thus, the number of equal consecutive corresponding digit is non-decrease with increase in (n) from 10000 to 100000.

At n = one billion ,

$\frac{1}{n}$ = 0.000000001 and $\frac{1}{n+0.1}$ = 0.000000001 is equal to at least 14 decimal places. Therefore, the functions $\frac{1}{n}$ and $\frac{1}{n+0.1}$ does not satisfy the requirement. This example show that mere proximity may not guarantee the existence of approximate functions that satisfy the conditions stated in the abstract.

Preliminaries of collapsing boundaries

Dosomah, Audu and Oriakhi (2013a) define unit point (x) of a subclass x^{Pn-1} e^{kx^n} of integrable functions as value $x_{p,n,k}(x) > 0$ such that

$\int_0^x x^{Pn-1} e^{kx^n} dx = 1$

The series

$\frac{1}{m}\sum_{n=4}^{\infty}\frac{m^{2n}}{2n-1} + \left(1-\frac{1}{m}\right)\sum_{n=4}^{\infty}\frac{m^{2n}}{2^n} - \sum_{n=4}^{\infty}\frac{m^{2n}}{2n}$

Where $m = \frac{K_{n+1}(n+1)}{1+2k_{n+1}}$ is the series associated with unit points of the $1-(n+1)-K_{n+1}$ exponential integral. The series is convergent when $m < \sqrt{2}$. The series is positive in the following ranges: $0 < m < 1, 1 < m < \frac{2n}{2n-1}$ and $\frac{2n}{2n-1} < m < \sqrt{2}$ and has a slow sum to infinity (less than 10), when n is one billion.

The unit point x_{n+1} is related to the series by $x_{n+1} = \sqrt[n+1]{\frac{1}{K_{n+1}}(S+P)}$ where (S) is the series associated with unit point and $P = \frac{m(2-m)}{2-m^2} - \frac{m^3}{6} - \frac{m^5}{20} - \frac{m^6}{24}$. Considering the significance and value elasticity of the series from simulation results of Dosomah, Audu and Oriakhi (2014), the authors opined that the series is a moderator constraining points to fit into the requirement of unit points. The authors (Dosomah et al, 2014), observe in the interval $1 < m < \frac{2n}{2n-1}$, that the K_{n+1} equivalent of the positivity conditions on m gave two functions:

$\frac{1+\sqrt{n+1}}{n+1}$ and $\frac{2n+\sqrt{4n^2+2n(n+1)(2n-1)}}{(n+1)(2n-1)}$. Approximately equal to 14 decimal places when n is one billion. Since the calculation require finding K_{n+1} values between boundaries, the co-incidence of values meant that within limits of calculation of K_{n+1} (at 3 significant figures) in the work, it is impossible to find a K_{n+1} to satisfy the positivity conditions in the interval $1 < m < \frac{2n}{2n-1}$. Since m > 0 and we cannot get K_{n+1} to make m > 1, the available option m < 1 was observed in the simulation result of Dosomah, Audu and Oriakhi (2014). Thus, the interval $1 < m < \frac{2n}{2n-1}$ had collapsing boundaries. Results from collapsing boundaries has been used as a blue print basis for the representation of life state as a decimal expansion in life state modeling, it has been related to life state as temptations of positivity. According to Dosomah, Audu and Edosomwan (2017), "if temptations of an activity turns one from positivity to do a wrong thing, the boundary has collapsed in that activity."

Statement of the problem

Following the discussion of collapsing boundaries, questions arising are:

1. Can one construct more pairs of approximate functions in the interval $1 < m < \frac{2n}{2n-1}$ that will be equal to at least 14 decimal places when n is one billion?
2. Can one construct pairs of that are equal to at least 14 decimal places in other intervals that did not have obvious collapsing boundaries?

Analysis of the problem

We (the authors of this paper) observe that the interval $1 < m < \frac{2n}{2n-1}$ that has collapsing boundaries has equality of upper and lower limits as (n) tends to infinity. Using this idea in the interval 0 < m < 1, we see that there is an abundance of collapsing boundaries in sub-intervals of 0 < m < 1 that may be used to generate pairs of function that tend to zero as (n) tends to infinity and are approximately equal to a high degree of accuracy when (n) is large. For example, since $\frac{2n}{2n-1} < 1$ and $\lim_{n\to\infty} \frac{2n}{2n-1} =$ *1, the Kn+1* equivalent of

$\frac{2n}{2n-1} < m < 1$ can be used to have collapsing boundaries within 0 < m < 1. In fact, classes of intervals of the form $\frac{2n}{2n-1} < m < 1$ where K is fixed and positive, will give collapsing boundaries within 0 < m < 1 Noting that:

$\frac{3n}{3n+K} - \frac{2n}{2n+k} = \frac{Kn}{(3n+k)(2n+k)} > 0$ where n and k are positive, implies

$\frac{3n}{3n+K} > \frac{2n}{2n+k}$. Thus the classes of nested intervals:

$\frac{2n}{2n+K} < m < 1, \frac{3n}{3n+k} < m < 1, \frac{4n}{4n+k} < m < 1, \ldots$ approaching their (1) upper bound will give collapsing boundaries within 0 < m < 1.

Construction and simulation results of pairs of approximate functions from collapsing boundaries

Steps to construction:

- Set a collapsing boundary at an (m) interval of positivity of a series associated with unit points of $1-(n+1)-k_{n+1}$ exponential intergral
- Find the (k)-equivalent of (m) from the series m – k relation.
- The k-equivalents of the lower and upper limits of m are the pairs of approximate functions.

Example 1

Using the sub-interval $\frac{2n}{2n+0.2} < m < \frac{2n}{2n+0.1}$ of 0 < m < 1, since $m = \frac{K_{n+1}^2(n+1)}{1+2K_{n+1}}$

$\frac{2n}{2n+0.2} < \frac{K_{n+1}^2(n+1)}{1+2K_{n+1}}$ and $\frac{K^2(n+1)}{1+2K_{n+1}} < \frac{2n}{2n+0.1}$

$\frac{2n}{2n+0.2} < \frac{K_{n+1}^2(n+1)}{1+2K_{n+1}}$ gives

$2n + 4nk_{n+1} < (2n+0.2)(n+1)\, K_{n+1}^2$

$0 < (2n+0.2)(n+1)\, K_{n+1}^2 - 4nk_{n+1} - 2n(1.31)$

The solution of the equation (2n+0.2)(n+1) $K_{n+1}^2 - 4nk_{n+1} - 2n = 0$ is

$K_{n+1} = \frac{4n \pm \sqrt{16n^2 - 4(2n+0.2)(n+1)(-2n)}}{2(n+1)(2n+0.2)}$

$K_{n+1} = \frac{4n \pm \sqrt{16n^2 + 16n(n+0.1)(n+1)}}{4(n+1)(n+0.1)}$

$$= \frac{4n \pm 4n\sqrt{1+\frac{(n+0.1)(n+1)}{n}}}{4(n+1)(n+0.1)}$$

$$= \frac{n}{(n+1)(n+0.1)}\left[1 \pm \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

The solution of the inequality is (1.31) is

$$= K_{n+1} > \frac{n}{(n+1)(n+0.1)}\left[1 + \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

And $K_{n+1} > \frac{n}{(n+1)(n+0.1)}\left[1 - \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$

Or $K_{n+1} < \frac{n}{(n+1)(n+0.1)}\left[1 + \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$

And $K_{n+1} < \frac{n}{(n+1)(n+0.1)}\left[\sqrt{1-\frac{(n+1)(n+0.1)}{n}}\right]$

These inequalities can be combined as

$$K_{n+1} > \frac{n}{(n+1)(n+0.1)}\left[1 + \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$$

$$\text{Or } K_{n+1} < \frac{n}{(n+1)(n+0.1)}\left[1 - \sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right] \qquad (1.32)$$

For the second part of the inequality $\frac{K_{n+1}^2(n+1)}{1+2K_{n+1}} < \frac{2n}{(2n+0.1)}$,

$K_{n+1}^2(n+1)(2n+0.1) < 2n + 4nk_{n+1}$

$K_{n+1}^2(n+1)(2n+0.1) - 4nk_{n+1} - 2n < 0$ (1.33)

The solution of the equation $K_{n+1}^2(n+1)(2n+0.1) - 4nk_{n+1} - 2n = 0$ is

$$k_{n+1} = \frac{4n \pm \sqrt{16n^2 - 4(n+1)(2n+0.1)(-2n)}}{2(n+1)(2n+0.1)}$$

$$k_{n+1} = \frac{4n \pm \sqrt{16n^2 + 8n(n+1)(2n+0.1)}}{2(n+1)(2n+0.1)}$$

$$k_{n+1} = \frac{4n}{2(n+1)(2n+0.1)}\left[1 \pm \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

$$k_{n+1} = \frac{2n}{(n+1)(2n+0.1)}\left[1 \pm \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

The solution of the inequality

(1.33) is $\frac{2n}{(n+1)(2n+0.1)}\left[1 - \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right] < K_{n+1} <$

$$\frac{2n}{(n+1)(2n+0.1)}\left[1 + \sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right] \qquad (1.34)$$

The solutions of inequalities (1.32) and (1.34) can be combined as

$$\frac{n}{(n+1)(n+0.1)}\left[1+\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right] < k_{n+1} < \frac{2n}{(n+1)(2n+0.1)}\left[1+\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$$

Let $A(n) = \frac{n}{(n+1)(n+0.1)}\left[1+\sqrt{1+\frac{(n+1)(n+0.1)}{n}}\right]$

And $B(n) = \frac{2n}{(n+1)(2n+0.1)}\left[1+\sqrt{1+\frac{(n+1)(2n+0.1)}{2n}}\right]$

Then A(n) $< K_{n+1} < B(n)$

Table (1.3a) shows the values of the two approximate functions A(n) and B(n) for n=1000, 10000, 100000, 1000000, 10000000, 100000000 and 1000000000

n	**A(n)**	**B(n)**	**Equality of decimal places**
1000	0.032620079	0.03262092	(6)
10000	0.010099938	0.010099964	(7)
100000	0.003172275969	0.003172276764	(8)
1000000	0.001000999949	0.001000999974	(10)
10000000	0.0003163277644	0.0003163277652	(11)
100000000	0.00010000000000	0.00010000000000	(14)
1000000000	0.0000316237766	0.0000316237766	(14)

Table (1.3a)

We observe from table (1.3a) that the pair of functions A(n) and B(n) tend to zero as n tends to infinity, the functions are non-decreasing in number of places of equal corresponding digits as n increase and at one billion they are equal to at least 14 decimal places with 9 non-zero digits to the right of decimal point. Thus the interval 0 <m < 1 that did not have obvious collapsing boundaries has sub-intervals of collapsing boundaries that satisfy the requirements.

Example 2

For the interval $1 < m < \frac{2n}{2n-1}$ that has obvious collapsing boundaries, we may use the k_{n+1} equivalent of sub-interval $\frac{2n}{2n-1} < m < \frac{2n}{2n-1}$ of $1 < m < \frac{2n}{2n-1}$ to obtain a pair of functions:

$C(n) = \frac{2n}{(n+1)(2n-0.1)}\left[1+\sqrt{1+\frac{(n+1)(2n-0.1)}{n}}\right]$

And

$D(n) = \frac{n}{(n+1)(n-0.1)}\left[1+\sqrt{1+\frac{(n+1)(n-0.1)}{n}}\right]$ to satisfy the requirement. Table (1.3b) shows the results

n	C(n)	D(n)	Equality of decimal places
1000	0.032622603	0.032623444	(5)
10000	0.010100015	0.01010004	(7)
100000	0.003172278355	0.003172279151	(8)
1000000	0.001001000024	0.001001000049	(10)
10000000	0.0003163277668	0.0003163277676	(11)
100000000	0.0001000100001	0.000100010001	(14)
1000000000	0.00003162377661	0.0000316237766	(14)

Table (1.3b)

Example 3

For the interval $\frac{2n}{2n-1} < m < \sqrt{2}$ that had no obvious collapsing boundaries we can get collapsing boundaries from sub-interval $\frac{2n\sqrt{2}}{2n+1} < m < \sqrt{2}$.

The k_{n+1} equivalent $E(n) < k_{n+1} < F(n)$

Where:

$$E(n) = \frac{2n\sqrt{2}}{(n+1)(2n-0.1)}\left[1 + \sqrt{1 + \frac{\sqrt{2}(n+1)(2n-0.1)}{4n}}\right]$$

And

$$F(n) = \frac{\sqrt{2}}{(n+1)}\left[1 + \sqrt{1 + \frac{\sqrt{2}}{2}\ (n+1)}\right]$$

Table (1.3c) shows the result

n	E(n)	F(n)	Equality of decimal places
1000	0.039025572	0.039026585	(5)
10000	0.012033694	0.012033724	(7)
100000	0.003774751928	0.003774752875	(8)
1000000	0.001190621544	0.001190621573	(10)
10000000	0.0003762017375	0.0003762017384	(11)
100000000	0.0001189348538	0.0001189348539	(14)
1000000000	0.00003760744516	0.0003760744516	(14)

Table (1.3c)

Thus E(n) and F(n) are pairs of functions that satisfy the requirements.

Observations

In all three tables (1.3a, 1.3b and 1.3c), we observe a pattern of first non-zero digits alternation. i.e. 3, 1, 3, 1, 3, 1, 3…. Is the same for the sequence of (n) values considered a steadiness in approach is indicated in the graph of unit points (fig 1) which he between 0.4 and 1.4.

The graph of unit point of the p – n – k class exponential integral when p = 1 for values of k= 0.1, 0.32, 1, 2, 3, 4, and n = 1 to 5

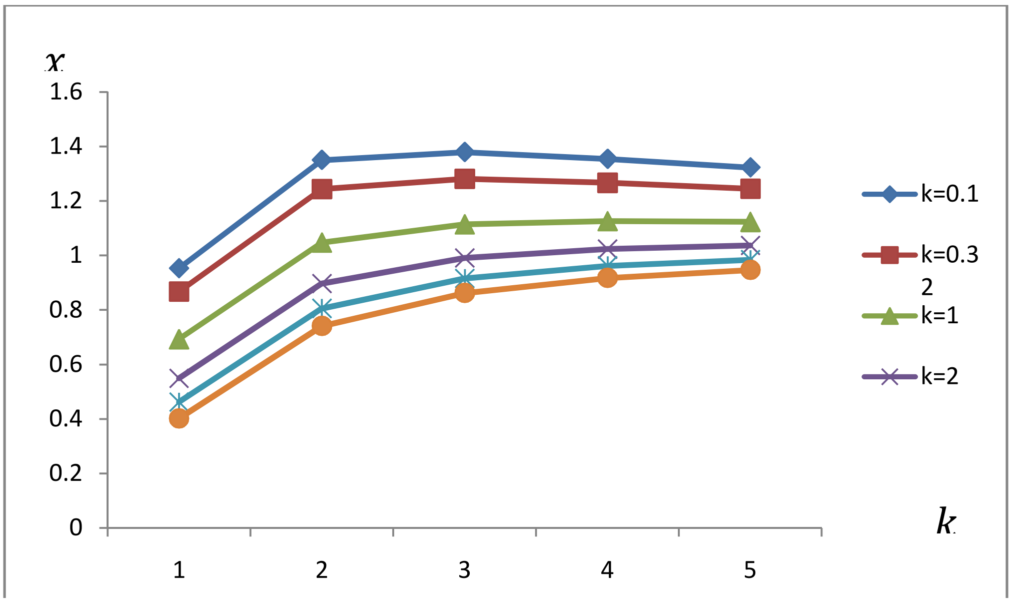

Fig (1) Dosomah 2014

The value characteristics of pairs of constructed functions from collapsing boundaries also shows steadiness in corresponding values and in approach to zero.

Research notes and principle of the construction

Unit points are simply values for which the integral of a certain function will be one. How does unit point relate to construction of pairs of approximate functions? The idea started from appreciating the beauty of the graph of unit points and the closeness of the graphs to one another. The next step was an attempt to investigate the structure of the function, hoping to find an explanation for the behaviour of the graph. Investigating the structure of the function by Taylor series expansion led to the discovery of series associated with unit points. Simulation results of calculating sum to infinity of the series to observation of collapsing boundaries in one of the positive intervals and an accidental discovery of two functions equal to 14 decimal places at one billion. Further research efforts led to clearer understanding of the process, construction of more collapsing boundaries and more approximate functions in positivity intervals with obvious and non-obvious collapsing boundaries and development of principles for the construction.

Principle of the construction

This principle was discovered by Dosomah, Audu and Edosomwan (2020) after studying Dosomah, Audu and Oriakhi (2014). It states:

If a convergent series has a transforming relationship and at least one interval; of positivity. If solving the inequality of the transforming relationship on collapsing boundaries points for which the series is positive gives a quadratic inequality with real roots, then collapsing boundaries of the series can be used to construct pairs of approximate function.

Conclusions

There are many collapsing boundaries of the range ($0 < m < \sqrt{2}$) of positivity of a series associate with unit points of the 1 – (n+ 1) - k_{n+1} exponential integral. Collapsing boundaries can be used to construct pairs of approximate functions that have limit zero as n tends to infinity, and satisfy the following conditions:

(1) From 1000 to one billion in steps of multiplier 10000 at least the first two digits from the first non-zero digit to the right of decimal point of the first function are respectively equal to the digits in corresponding positions of the second function.

(2) The number of equal consecutive corresponding digits in the first set of consecutive corresponding digits to the right of decimal point in both functions increase with increase in (n) in the given range.

(3) At n equal one billion, the functions are equal to at least 14 decimal places

Recommendations

Based on the research findings in this paper, the following recommendations are proffered:

- Research should be intensified to apply the principle of the construction to construct further approximate functions from collapsing boundaries of suitable convergent series.
- Researcher should be encouraged to explore processes in novel manner towards breaking new grounds.

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