THE MAJOR SCHOOLS OF THE CLASSICAL PERIOD: THE MATHEMATICAL INVOLVEMENT

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Abstract

The theme of this paper is "the major schools of the classical periods: the mathematic involvement". The highlights are centred on Mathematics involvement in Ionian school, Mathematics involvement in Pythagorean School, Mathematics involvement in Eleatic school, Mathematics involvement in Sophist school, Mathematics involvement in Platonic school, Mathematics involvement in the school of Eudoxus and Mathematics involvement in Aristotle school. The paper concludes with recommendations.

The word Philosophy was derived from two Greek words, "Philo" and "Sohia" which combine to give its raw form "Philosophia" meaning "love of wisdom" or "love of knowledge". But thereafter many scholars tend to look at philosophy in different perspective and so the varying definitions. The thinking further segmented the discipline into different subject units of which mathematics is one, which is the centre of this paper discussion. But it is unworthy while discussion philosophy of mathematics without having a look at some definitions of philosophy and so below are some of these definition. Enukoha, Asuquo and Inaja (2004) see philosophy as someone's attitude to life. Mibit (1982) perceived it as the attitude of mind, logic and perception being the manner an individual thinks, acts or speak in different situation of life. Sharma and Hyland (1991) define philosophy as a process of asking questions about the world, man's place in the world and all aspects of human activities and experiences. The philosophical perception of mathematics by philosophers is also worth mentioning. Ernest (1986) in Foin (1997), define philosophy of Mathematics Educaiton as an inquiry into the nature of mathematics, its reflection and characteristics concepts, methods,

insight or truth and values. While Zheng (1994) in Foin (1997) held that philosophy of mathematics education should be concerned with the questions, what is mathematics?, why should we search or learn it? And how should we teach or learn it?
Proclus in Kline (1972) philosophized mathematics and declares “this, there, is mathematics: she reminds you of the invisible form of the soul; she gives life to her own discoveries; she awakens the mind and purifies the intellect; brings light to our intrinsic ideas; she abolishes oblivion and ignorance which are ours by birth”. and for more of these philosophical details philosophy of mathematics on this paper will focus on the seven major schools that generated the creation of classical period in Greek mathematics which started from 600-300Bc. These schools are: the Ionian, Pythagorean, Eleatic, Sophist, Platonic, Eudoxus and Aristotle Schools.

IONIAN SCHOOL

This school was founded by Thales in C.640 B.C in Miletus and lasted till C.540 B.C. Some of his students where Anaximander (C.610 – C.428 B.C) and Pythagoras (C.585 – C.500 B.C). He was born and lived in Miletus and later traveled and settled in Egypt as a businessman. In his business activities, it was recorded that he learnt much about Egyptian mathematics during that period. He predicted an eclipse of the sun in 585 B.C. but because knowledge about astronomy was faint in the life of the people they disbelieved him and disputed it. One other contribution he made was the computation of the heights of Pyramid. He did this by comparing the shadows of a stick with a known height and the shadows of the Pyramids that were cast along side with the stick. He was also able to calculate the distance of ship form shore with similar knowledge acquired from triangles computations. Although other authors disprove the idea, he was also being attributed the founder of abstract mathematics and made deductive proof on some theorems.

Other contributions of Thales include the discovery of the attractive power of magnets and of static electricity.

PYTHAGOREAN SCHOOL

Pythagoreans after the reign of Thales found his own school in a Greek settlement in Italy called Croton. He was born on the Island of Samos, a place close to the coast of Asia Minor. After leaving his master Thales, he went to Egypt and Babylon, where he had some knowledge of mathematics and mystical doctrines which enabled him to open a school on religious, scientific, and philosophical brotherhood. One of the great Greek contributions to the concept of mathematics was the conscious recognition and emphasis of the fact that mathematical entities, numbers and geometrical figures are abstract ideas, entertained by the mind which was sharply distinguished from physical objects or pictures and it becomes a field of study. This great Greek contribution was attributed to the scholars of Pythagorean School.

The early Pythagoreans considered numbers as units that composed individual objects. They saw numbers as the essence of the universe and that was why Aristotle also declares that the Pythagoreans regarded numbers as the ultimate components of real, material objects; in that numbers did not have a detached existence apart from objects of sense. Before and after his death most mathematical contributions are attributed to him and his scholars. These include introduction of pure mathematics, Pythagorean and incommensurable ratio.

The Pythagoreans usually take numbers as dots in sand or as pebbles. It was on that basis they considered numbers like 1, 3, 6 and 10 as triangular because the corresponding dots could be arranged as shown in figure 1.

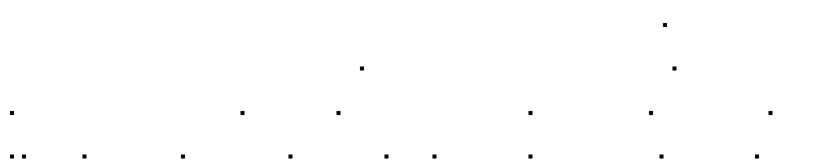
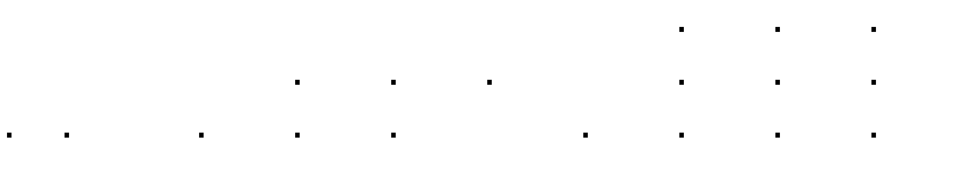


Fig1: Triangular Dots



And that these triangular numbers could be gotten through the following formula n(n+1). In the same vein numbers like 1, 4, 9, 16, etc were called square numbers because dots were arranged to form squares with these numbers as shown in figure II. And any square number could be gotten through the following formula $(n+1)^2$

Fig III: Square Numbers

The Pythagoreans also used the dots to get polygonal numbers such as pentagon, hexagon and others. Unlike the triangular and square numbers, they did not have a single formula in getting these polygonal numbers. Two polygonal numbers are shown in figure III.

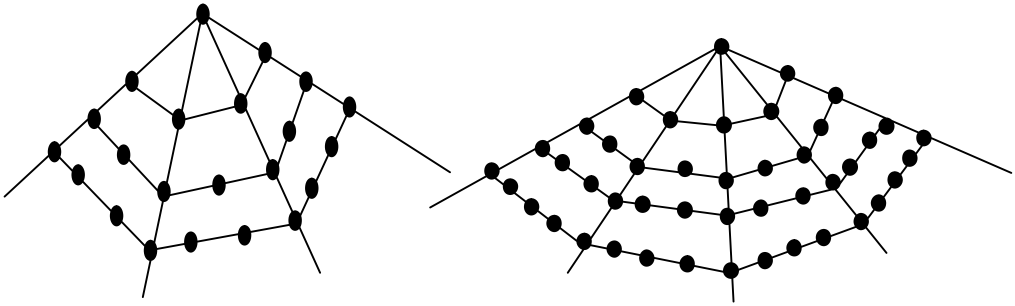


Fig III: Polygonal Numbers

The Pythagorean Theorem was the contribution of Pythagorean School which states that in a right angled triangle the square on the side subtending the right angle is equal to the sum of the squares of the two sides forming the right angles. See its diagrammatical presentation below.

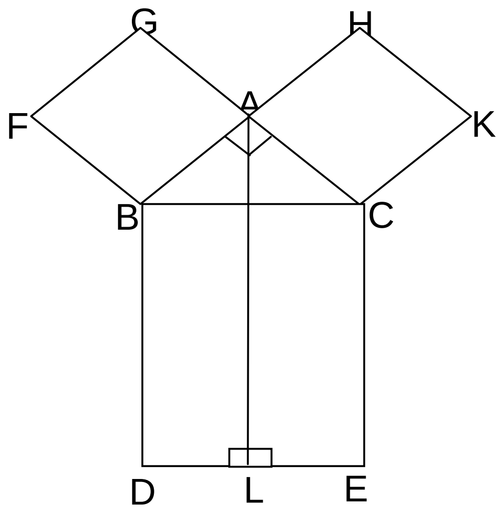


Fig iv: Pythagorean Theorem

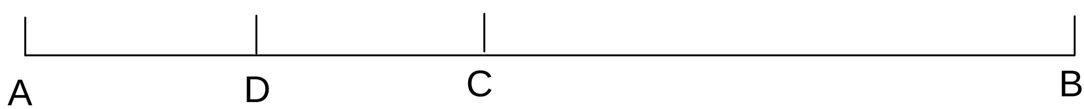
The area of the figure BCED is equal to the sum of the areas of BFGA and CAHK.

THE ELEATIC SCHOOL

This was founded by a philosopher called Zeno who lived in the southern Italian city of Elea, hence the name Eleatic school. Zeno School of Philosophy propounded that space and time are infinitely divisible (that is to say motion is continuous) and believe further that space and time are made up of indivisible small intervals (that motion is a succession of minute jerks). In defense of the two theories he raised four paradoxes. He directed the first two to the first theory while the last two for the second theory. The paradoxes are the Dichotomy; Achilles and other Tortoise; the Arrow and the Stadium or the moving Rows. Zeno proposed these paradoxes in support of his master Parmenides, who is of the view that motion or change is impossible.

Dichotomy

Zeno first proposition states that motion is nonexistence. He made his proposition on the ground that any object in motion must arrive at the half way stage before it arrives at the goal. He demonstrated this with a diagram shown below. In his argument he stated that for one to move he must first arrive at the point D. and so forth. So based on the assumption that space is infinitely divisible, he concluded that a finite length contains an infinite number of points, therefore it is impossible to cover event of a finite length in a finite time.



Achilles and the Tortoise

Zeno postulated that the slowest moving object cannot be overtaken be the fastest since the pursuer must first arrive at the point the pursued started, so necessarily the slower one is always a head. Zeno's proposition could be viewed in three perspectives. One, the situation could be possible if both objects are moving with the same speed. Two, the situation could still be possible, if the first object is moving with a faster speed. Third the situation could not be possible, if the second object (the purser) is moving with a faster speed. Here the pursuer will overtake the pursued but still it depends on the distance.

Arrow

Zeno here believed that when an arrow is short into the air, that moving arrow is at a standstill. His assumption was that time is made up of instants. But as Aristotle rightly pointed out, Zeno did not think of indivisible units of time that is why his proposition failed. On the other hand he may be correct if he is talking of the arrow moving at a given height into a garget horizontally.

Stadium or the Moving Rows

This paradox started that a set of bodies moving on a race-course and passing another set of bodies equal in number and moving in the opposite direction, the one starting from the end and the other from the middle, both are moving in equal speed. This he concludes by saying that half of the time is equal to double the time. “Zeno” position is justified if the postulation started in line with the following demonstration. That supposing there are three rows of soldiers, row A, row B and Row C and that at the smallest unit of time, row B moves one position to the left and at the same time row C moves one position to the right. So the distance between B and C doubles the distance between A and B, A and C, which implies half the unit of time. See the figure below.

A	.	.	.	.	.	A	.	.	.	.
B	.	.	.	.	.	B.	.	.	.	
C	.	.	.	.	.	C		.	.	.
	.									

Fig vi: Rows of Soldiers

From the forgoing postulation, Zeno must have intended to point out that speed is relative. C’s speed relative to B is not C’s speed to A. That it will take double the time of moving to B from C than to A.

THE SOPHIST SCHOOL

Here emphasis was given to abstract reasoning and the scholars thought it wise to extend this lofty idea into the entire universe, having man as the target point. It was the first school established in Athens and it embraced learned teachers of grammar, rhetoric, dialectics, eloquence, morals, geometry, astronomy and philosophy. One of their chief objectives was to use mathematics in understanding the functioning of the universe. The school has mainly faced with the solving of three famous construction problems: to construct:

i. A square equal in area to a given circle
ii. The sides of a cube whose volume is double that of a cube given edge and
iii. To trisect any angle.

The above constructions are to be performed with straight edge and compass only. The construction directive came from the oracle which was consulted as a result of pestilence suffered by the Delians. The oracle advised that an altar doubling the size of the existing one should be constructed to avert the disease but that did not augur well with them because doubling the side would not double the volume; so they went for further consultation. The earliest known attempt to solve any of the three famous problems was made by the Ionian Anaxagoras, who tried working on the squaring of the circle while in prison; but never concluded. Another famous attempt was made by Hippias of Elis. He was a leading sophist then and a contemporary of Socrates. In his attempts to trisect an angle, Hippias invented a new curve and called it quadratrix and he constructed it in the following order: Let AB rotates clockwise about A at a constant speed the position AD. At the same time let BC move downward parallel to itself at a uniform speed to AD. Suppose AB reaches AD' as BC reaches B'C'. Let E' be the intersection of AD' and B'C. The E' is a typical point on the quadratrix BE'G is the final point on the quadratrix. See diagram below.

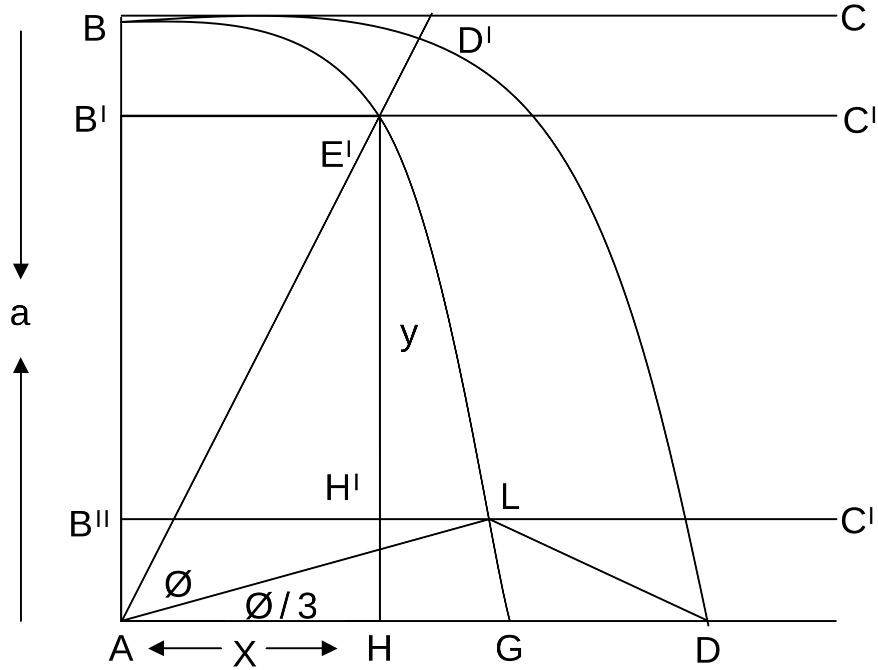


Fig vii: Quadratic Generation

The equation of the quadratic in terms of rectangular Cartesian coordinates is E'H/BA which equals ⍰/π/2. This happens when AD' reaches AD in some fraction of time t/T of the total time T, that AB takes to reach AD. Since AD' and B'C move at constant speeds, B'C' covers that part E'H of BA as AD' of BA in the same fraction of the total time, so the equation of quadratic becomes the form state above.

Transforming the above equation, denote E'H by y and BA by a and it will become

$\frac{}{\pi/2} = \frac{y}{a}$ or y = a. ⍰ $\frac{2}{\pi}$ ……………..……………… (1)

But if AH = x, then

θ = arc tan $\frac{y}{x}$ …………………………………..… (2)

And replacing θ in eqn (1), we have

y = $\frac{2a}{\pi}$. arc tan $\frac{y}{x}$ or y = x tan $\frac{y}{2a}$ ………….……. (3)

To show that the curve if constructible, could be used to trisect any acute angel. Let θ be such an angle in Fig VII. Then trisect y so that E'H = 2H'H. Draw B" C" through H' and let it cut the quadratrix in L. Draw AL. Then L$\hat{A}$D = θ/3. From the argument which led to (1), the following is deduced.

L$\hat{A}$ D = $\frac{H'H}{\pi/2a}$ or $\frac{L\hat{A}D}{\pi/2a}$ = $\frac{y}{3a}$ OR L$\hat{A}$D = $\frac{\pi}{2}$. $\frac{Y}{3a}$ ……………..… (4)

But by (1) we have $\frac{}{\pi 2}$ = $\frac{y}{a}$

Hence L$\hat{A}$D = $\frac{\pi/2}{3}$ $\frac{}{\pi/2}$ = $\frac{}{32}$ ……………………………………….…… (5)

Another problem that was not so famous, like the discussed three famous construction problems, was the construction of regular polygons of seven and more sides; which led to another search of knowledge in Greek mathematics, during the time of sophist school.

THE PLATONIC SCHOOL

The school succeeded the sophists in the leadership of mathematical search of knowledge. Plato got his knowledge from his fore runners, Theodorus of Cyrene in North Africa (born in 470 B.C) and Archytas of Tarentum in Southern Italy (born in 428-347 B.C). Both were Pythagoreans. Plato founded his Academy also in Athens and the school grew up to became liked a Modern University, because it has grounds, buildings, students, and formal courses taught by himself and his aides. The school favored the study of Mathematics and philosophy but the main Mathematics centre was later moved to Alexandria about 300 B.C. while the philosophy remained with the Academy throughout the Alexandria period.

Plato, who was one of the most informed men in his days, was not a Mathematician, yet because of his enthusiasm for the subject and his belief of its importance in philosophical matters and again for the understanding of the universe through it, encouraged mathematicians to pursue its knowledge.

It was on that struggle, almost all the important mathematical works of the fourth century was done by his friends and pupils. Plato was more concerned in perfecting and improving what was known as mathematical knowledge. He went further to say that "numbers and geometrical concepts have nothing materials in them and are distinct form physical things; the concepts of it are independent of experience and have a reality of their own; they are discovered, not invented or fashioned ". Still for the love of mathematics, he went again to philosophize that "and do you not know also that although they further make use of the visible forms and reason about them, of the ideals which they resemble… But they are really seeking to behold the things themselves, which can be seen only with the eye of the mind".

Plato sees mathematical ideas as abstract ideas and he likened these abstract ideas to goodness and justice in philosophy. Hence he concluded that mathematics is the preparation for knowledge about the ideal universe. Plato and others sharply differentiate between the world of ideas and the world of things. Plato particularly believed that the perfect ideas of physical objects are reality. To the Platonists the world of ideals and relationship is permanent, ageless, incorruptible and universal, while the physical world is an imperfect realization of the ideal world and is subject to delay, hence they believe that the ideal world alone is worthy of study.

Two types of methodology were again credited to the Platonists by Proclus and Diogenes Laertius in the 3rd cent. A.D. The first is the method of analysis, where what is to be established is regarded as known and the consequence is deduced until a known truth or a contradiction is reached. If a contradiction is reached then the desired conclusion is false. If a known truth is reached then the steps are reversed and the proof is made. The second is the method of reduction absurdum or the indirect method. Plato indeed affirmed the desirability for a deductive organization of knowledge; hence he was attributed to be the first to systematize the rule of rigorous demonstration.

The most significant discovery by the Platonic school was the conic sections. The discovery was believed to have emanated from the famous construction problems. Menaechmus whom the discovery was attributed saw the conic sections in three different forms while working on the famous construction problems. These three forms of cones are right-angled, acute-angled and abtuse-angled. See the cones below.

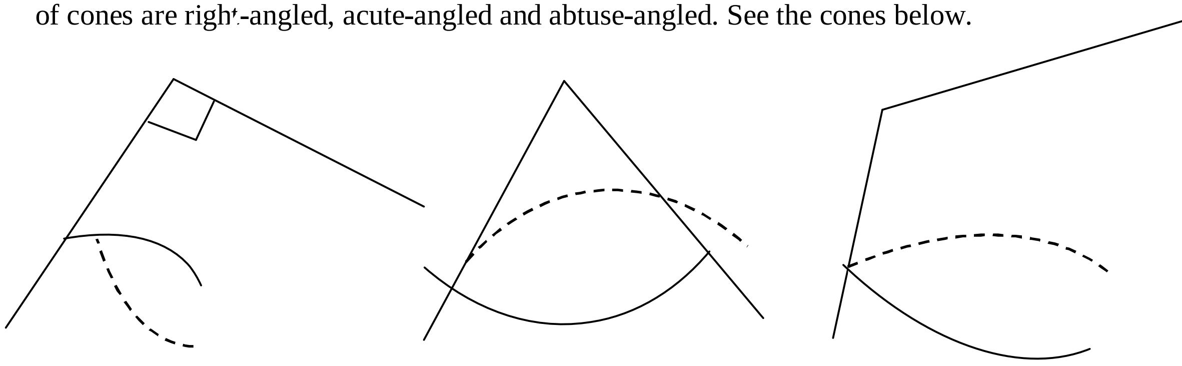


Fig viii: The three types of Cones

One more study that was attributed to Platonic school was the work on incommensurables. Theaetetus who was the investigator of Cyrene's works on square roots and irrational went ahead and worked on higher types of irrationals and classified them.

THE SCHOOL OF EUDOXUS

Eudoxus was born in Cyzicus in Asia Minor about 480 B.C. He studied under Archytas in Tarentum and later founded a school at Cyzicus in Northern Asia Minor. His first great contribution to mathematics was a new theory of proportion. The discovery of more and more irrationals or incommensurable ratios made it necessary for the Greek to face these numbers at the time of this school. There was this question of how proofs, areas, and volumes be extended to incommensurable ones? In an answer to this question, Eudoxus introduced the notion of a magnitude which stood for entities such as lien segments, angels, areas, volume and time, which could vary continuously. He defined a ratio of magnitude and a proportion as an equality of two ratios, to cover commensurable and incommensurable ratios. However he did not assign numerical values to the ratios, rather the concepts were tied to geometry.

The Eudoxian solution to the problem of treating incommensurable lengths or the irrational number actually reversed the emphasis of previous Greek Mathematics. His theory of not assigning numerical values to lengths of line segments, sizes of angels and other magnitudes and to ratio of magnitudes had enabled the Greek Mathematicians to make tremendous progress in geometry by supplying the necessary logical foundation for incommensurable ratios, yet it had several unfortunate consequences. Three among the unfortunate consequences are

(1) It has forced a sharp separation between number and geometry, for only geometry could handle incommensurable ratios.

(2) It also drove Mathematicians into the ranks of the geometers and geometry became the basis of almost all the rigorous mathematics for the next two thousand years.

(3) We still speak of x^2 as x square and x^3 as x cube instead of x to second or x third power. It had been so pronounced because the magnitudes x^2 and x^3 had only geometric meaning to the Greek.

Since Eudoxus undertook to provide the precise logical basis for these ratios, he most likely saw the need to formulate axioms and deduced consequences one by one, so that no mistake would be made with these unfamiliar and troublesome magnitudes. What actually informed him in taking this work to this level was the belief of the Greeks at that time. The Greek sought truths and had decided on deductive proof and so they had to obtain axioms that were themselves truths. They did find statements whose truth was self-evident to them, though the justifications given for accepting axioms as disputable truth, varied. This need to work with incommensurable ratios also undoubtedly reinforced the earlier decision to rely on deductive reasoning for proof.

The powerful Greek method of establishment the area and volumes of curved figures, which they called method of exhaustion, was also attributed to Eudoxus. It was the first step in calculus but doesn't use an explicit theory of limits. With that method, he proved the following: that the areas of two circles are to each other as the squares of their radii, the volumes of two spheres are to each other as the cubes of t heir radii, the volume of a pyramid is one–third the volume of a prism of the same base and attitude, and the volume of a cone is one-third the volume of the corresponding cylinder.

Indeed the Eudoxus philosophy of incommensurable ratios have realigned the Pythagorean philosophy which believes that a line is made up of a finite number of points (which they identified with physical particles) but could not be the case for a length such as $\sqrt{2}$.

ARISTOTLE AND HIS SCHOOL

He was born in 322 B.C and died in 384 B.C at a city in Macedonia called Stagira. He founded his own school called Lyceum, which had a garden, a lecture room and an altar to the Muses. But before he had his own school he was a tutor to Alexander the Great. His contributions were on mechanics, physics, mathematics, logic, meteorology, botany, psychology, zoology, ethics, literature, meta-physics, economics and many other fields. But for all the contributions he had made, there is no one book on mathematics. He only applies it in variety of places and uses it to illustrate a number of points. Though Aristotle did not contribute significantly to new mathematical results, his views on the nature of mathematics and its relations to physical world were highly influential. He place mathematics under theoretical science, which he believe, seek exact Truth. Aristotle also believed that the universe is made of physical objects or concrete matters such as hardness, softness, heaviness, lightness, sphericity, coldness and warmness and that numbers and geometrical forms were also properties of real objects. Thus mathematics deals with abstract concepts, which are derived from properties of physical bodies.

Aristotle discusses definition. His notion about definition is that it must be in terms of something prior to the thing defined. The existence of defined things has to be proved except in the case of a few primary things such as point and line, whose existence is assumed along with axioms. Thus one can define a square but if the properties demanded in the definition are not physically seen, then such a figure may not exist. Leibniz's regular polyhedron with ten faces was given as an example. This figure can be defined but it does not exist. The method he adopted to prove the existence of an object was construction.

Aristotle also treated the fundamental problems of how points and line can be related. A point he said is not divisible and has position. But no accumulation of points, however far it may be carried, cannot give anything divisible, whereas a line does. He liken "point" to "now" in time. Now is indivisible, not continuous and not part of time, so also a point is to line. A point may be an extremity, beginning or divider of a line but is not a part of the line. It is only through motion; a point can generate a line and thus be the origin of magnitude. He also argued that a point as no length and so if it forms a line, such a line has no length. The substance of his doctrine is that points and numbers are discrete quantities and must be distinguished from continuous magnitudes of geometry. He equally argued that here is no continuum in Arithmetic and preferred it to geometry because it is more accurate, numbers lend themselves to abstraction more readily than the geometric concepts. He also considers arithmetic as prior to geometry because the number three (3) is needed to consider a triangle, be it the sides or angles or apexes.

Aristotle also discussed infinity, according to him, it is only the potentially infinite that exist. He cited positive integers as an example of potentially infinite number s, that 1 can add to any number and get a new one but that infinite set as such does not exist. Another major achievement of Aristotle was the founding of the science of logic. His basic principles of logic were the law of contradiction, which says a proposition cannot be both true and false, and the law of excluded middle, which maintains that a proposition must be either true or false. He called them the heart of the indirect method of mathematical proof. He used mathematical examples, taken from contemporary texts to illustrate his principles of reasoning. Though the science of logic was derived from mathematics, logic eventually came to be considered independent and Aristotle regards it as preliminary to science and philosophy, (Kline 1972:51-54).

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