COMPARISON OF NON-INFORMATIVE PRIORS IN BAYESIAN MODEL AVERAGING

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Abstract

A new g-prior has been proposed called averaging-g-prior (av-g-prior), and it is used to compare some popular g-priors in literature, namely, Uniform Information Prior (UIP), Hannan-Quinn Criterion (HQ) and Benchmark Prior, in the light of different model priors. The results show that the proposed prior produces posterior means and posterior standard deviation similar to those given by the benchmark prior for most of the variables used in the analysis. However, in some variables, the proposed prior produced a smaller posterior standard deviation than that produced by benchmark prior. Furthermore, the proposed prior gave a smaller posterior standard deviation for all the variables in the analysis than those produced by the UIP and HQ, indicating dominance. In addition, the proposed prior shows a higher posterior inclusion probability in most variables than the benchmark prior. We recommend therefore that the av-g-prior be used in Bayesian model averaging when no prior information about the subject of analysis is available.

Keywords: Bayesian model averaging, av-g-priors, model priors, posterior standard deviation, Markvov Chain Monte Carlo

Bayesian model averaging as developed by Leamer (1978), Raftery (1998), Madigan and Raftery (1994) has become a widely accepted method of analysis as its strength lies in accounting for model uncertainty. Several works have been done in the last decade using Bayesian model averaging method. Annest, Bumgarner, Raftery and Yeung (2009) used the iterative Bayesian model averaging (BMA) method in applying survival analysis to microarray data to determine a highly predictive model of patient's time to event (such as death, relapse or metastasis) using a small number of selected genes. Bayesian model averaging (BMA) has also been applied in estimating the association between air pollutants and fatal health outcomes, Fang, Li, Kan, Fang and Cao (2016). Kaplan and Chen (2014) investigated the use of Bayesian model averaging in propensity score analysis for quasi-experimental or observational studies. Eicher, Pagageorgiou and Raftery (2011) used predictive performance in Bayesian model averaging to assess the prior distributions. Sloughter, Gneiting and Raftery (2013) also extended Bayesian model averaging (BMA) methodology to use bivariate distributions to provide probabilistic forecast of wind vector.

However, one of the major challenges in the implementation of Bayesian model averaging is that of specifying the prior probability of the parameters on each model and the prior probability of each model. Where there is prior information about the subject of the analysis a prior distribution can easily be formed and used, otherwise a non-informative prior is being suggested.

Several non-informative priors have been suggested in literature, among them are the following: the Uniform Information Prior (UIP), which is approximated by the Schwarz criterion (BIC), Kass and Wasserman (1995); the Risk Inflation Criterion (RIC), Foster and George (1994); the Hannan-Quinn Criterion (HQC), Hannan and Quinn (1979); the Benchmark Prior of Fernandez, Ley and Steel (2001a).

The Uniform Information Prior having been criticized as being too conservative, Fernandez et al (2001a) suggested that prior should be large enough as to minimize the effect it will have on the result so as to keep the result close to ordinary least squares coefficients. However, Ciccone and Jarocinski (2010) demonstrated that under noisy data such a large prior may not accommodate the noise and this consequently may lead to over-fitting.

In this study, we sought to compare the different priors in the light of some model priors with a newly proposed prior.

Bayesian Model Averaging

Bayesian Model Averaging accounts for the uncertainty involved in model selection. This approach is to make inference from a posterior distribution defined on the model space. If Δ is the quantity of interest such as a future observation or the utility of a course of action, and $M=(M_1,...,M_l)$ denotes the set of all models considered, then the posterior distribution of Δ given the data D is

$$P(\Delta \mid D)=\sum_{i=1}^{l} P(\Delta \mid M_i, D)P(M_i \mid D) \quad (1)$$

This is an average of the posterior distributions under each of the models considered weighted by their corresponding posterior model probabilities. The posterior probability of the model M_i is given by

$$P(M_i \mid D)=\frac{P(D \mid M_i)P(M_i)}{\sum_{j=1}^{l} P(D \mid M_j)P(M_j)} \quad (2)$$

Where $P(M_i)$ is the prior probability that M_i is the true model, and $P(D \mid M_i)=\int P(D \mid \theta_i, M_i)P(\theta_i \mid M_i)d\theta_i$ is the marginal likelihood of M_i , θ_i is the vector of parameters of model M_i , $P(\theta_i \mid M_i)$ is the prior density of θ_i under model M_i, $P(D \mid \theta_i, M_i)$ is the likelihood.

The posterior mean and variance are as follows:

$$E(\Delta \mid D)=\sum_{i=0}^{l} \hat{\Delta}_i P(M_i \mid D) \quad (3)$$

$$\operatorname{var}(\Delta \mid D)=\sum_{i=0}^{l} \left(\operatorname{var}[\Delta \mid D, M_i]+\hat{\Delta}_i^2\right) P(M_i \mid D)-E(\Delta \mid D)^2 \quad (4)$$

where $\hat{\Delta}_i=E(\Delta \mid D, M_i)$ Raftery (1993) and Draper (1995).

Bayesian model averaging has been found to predict substantially better than any single model if performance is measured using the logarithmic predictive score, Fernandez, Ley and Steel (2001b), Ley and Steel (2009).

Some g-priors in Literature

The uniform information prior (UIP) which is closely approximated by the Schwartz (Bayes information) criterion sets $g = n$. Risk inflation criterion sets $g = k^2$

According to Fernandez, Ley and Steel (2001a), two functional choices of g were examined. First, the dependence on the sample size n and on the number of predictors k. Considering the result under

$$g = \frac{w_1(k_i)}{w_2(n)} \text{ with } \lim_{n\to\infty} w_2(n) = \infty$$

As the sample size increases the precision of the prior becomes a smaller fraction of that of the sample and reduces to zero as n tends to infinity. We let g depend on a function $w_1(k_i)$. The results are summarized in the theorem below (Fernandez, Ley and Steel, 2001a):

Theorem 1:

Consider the Bayesian model $y = \alpha + Z_i\beta_i + \varepsilon$ together with the prior densities given by

$$P(\sigma) \propto \sigma^{-1} \ , \ P(\alpha) \propto 1 \ , \ P(\beta_i \mid \sigma, M_i) = f\left(\beta_i \mid 0, \sigma^2 \left(gZ_i{}'Z\right)^{-1}\right) \quad 5$$

and any prior on the model space M in

$$P(M_i) = P_i, \quad i = 1, \ldots, 2^k \text{ with } \beta_i > 0 \text{ and } \sum_{i=1}^{2^k} P_i = 1 \quad 6$$

Then under the assumption that there is a true model M_s in M that generates the data, the condition

$$\lim_{n\to\infty} \frac{w_2'(n)}{w_2(n)} = 0 \quad 7$$

together with either

$$\lim_{n\to\infty}\frac{n}{w_2(n)}\in[0,\infty)$$ or $w_1(\cdot)$ is a non decreasing function that

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ensures that the posterior distribution of the models is consistent.

Two information criterion that have been used for model selection in time series are the Schwarz or Bayes information criterion and the Hannan-Quinn criterion. They are in the form

$$S_{is}=\frac{n}{2}\ln\left(\frac{y'M_{X_s}y}{y'M_{X_i}y}\right)+\frac{k_s-k_i}{2}\ln(n)$$

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$$HQ_{is}=\frac{n}{2}\ln\left(\frac{y'M_{X_s}y}{y'M_{X_i}y}\right)+\frac{k_s-k_i}{2}C_{HQ}\ln\ln(n)$$

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Where

$$y'M_{X_i}y=y'y-y'X_i\left(X_i'X_i\right)^{-1}X_i'y$$

These two criteria are consistent provided $C_{HQ}>2$.

Fernandez *et al.* (2001a) after examining different choices of g depending on the sample size or the number of predictors recommends $g=\max\left(n,k^2\right)$. Hannan-Quinn (1979) suggests $g=(\ln n)^3$ where $C_{HQ}=3$.

We therefore propose a g-prior called averaging-g-prior (av-g-prior) that shares in the properties of the benchmark prior of Fernandez *et al.* (2001a) and Hannan-Quinn criterion. This prior sets

$$g=\frac{1}{2}\left[\max\left(n,k^2\right)+(\ln n)^3\right]$$

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Model Priors

The uniform model prior was suggested by Raftery (1998), and George and McCulloch (1993). It assigns equal prior probability to all 2^k models, so that $P(M_i) = 2^{-k}$. There are other model priors such as the binomial model prior proposed by Mitchell and Beauchamp (1998)

$$P(M_i) = \theta^{\delta_i}(1-\theta)^{k-\delta_i}$$

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which places the prior probabilities of a model size as a product of inclusion and exclusion probabilities (Zeugner and Feldkircher, 2015) and also the Beta-binomial model prior. Ley and Steel (2009) were of the opinion that θ being a random variable should be drawn from a Beta distribution, Eicher et al (2011). However, in this study, we have used the uniform model prior.

Results and Discussion

In this work, data were obtained from Central Bank of Nigeria statistical bulletin having gross domestic product (gdp) as the response variable and 19 predictor variables which are described as follows: industrial output (indQ), money supply (ms), gross fixed capital formation, credit to private sector (cps), recurrent expenditure (recEx), balance of payment (bop), savings (sav), stock market capitalization (smc), external reserve (extR), external debt (extDt), income tax (incTx), unemployment (uemp), financial deepening (fd), oil price (oilp), domestic debt (domD), inflation (inf), exchange rate (excr), capital expenditure (capEx), lending rate (lr).

The Markov Chain Monte Carlo (MCMC) birth-death (bd) method was employed. Each of the g-priors considered is used with the different model priors, namely, uniform model prior, binomial model prior and the beta-binomial model prior. The results obtained were as follows:

Table 1: Post mean (uniform model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	2.47073	3.10135	2.83567	3.33687
ms	0.52980	1.32708	0.64333	0.80503
gfcf	0.00185	0.00207	0.00174	0.00197
cps	-0.35141	-0.74796	-0.33681	-0.39109
recEx	2.24520	0.97185	1.70952	0.64950
bop	-0.00009	-0.00012	-0.00010	-0.00009
sav	0.49454	0.22626	0.30649	0.31711
smc	0.06799	0.03053	0.05649	0.03681
extR	0.03138	0.01147	0.02490	0.00924
extDt	-0.01315	-0.01639	-0.01542	-0.01764

incTx	0.00060	0.00016	0.00035	0.00021
uemp	-10.59252	-6.45127	-10.60702	-5.90529
fd	-19.51910	-11.34953	-12.85910	-6.44810
oilp	0.36583	0.13793	0.44556	0.17378
domD	0.44885	-0.00156	0.32533	0.06317
inf	-0.48133	-0.17379	-0.39520	-0.22241
excr	-0.96885	-0.30274	-0.76444	-0.56742
capEx	0.01651	0.00544	0.00305	-0.03395
lr	-0.48133	0.00883	-0.13159	-0.09504

Table 1 shows the posterior mean of some variables, namely, gfcf, bop, smc, extR, extDt, incTx, inf, oilp obtained using the benchmark prior are similar to those obtained using the proposed prior.

Table 2: Post mean (binomial model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	2.58255	3.36806	3.11275	3.52773
ms	0.57699	1.22494	0.35546	0.61280
gfcf	0.00189	0.00163	0.00128	0.00192
cps	-0.27112	-0.62284	-0.17051	-0.21421
recEx	2.18709	0.49936	1.62192	0.37935
bop	-0.00008	-0.00007	-0.00007	-0.00007
sav	0.22907	0.27248	0.35206	0.29204
smc	0.05294	0.03389	0.04815	0.03138
extR	0.03371	0.00644	0.02782	0.00701
extDt	-0.00792	-0.01151	-0.01133	-0.01578
incTx	0.00048	0.00013	0.00033	0.00015
uemp	-6.93440	-4.02646	-8.61560	-3.78603
fd	-13.89111	-8.85571	-0.27723	3.25200
oilp	0.28145	0.10430	0.34157	0.16352
domD	0.33083	0.02639	0.24067	-0.00349
inf	-0.30011	-0.13022	-0.42824	-0.09307
excr	-0.99871	-0.27019	-0.77129	-0.25816
capEx	0.06858	-0.03759	-0.01909	-0.03685
lr	-0.24803	-0.01647	-0.58311	-0.04296

It is observed from Table 2 that the variables gfcf, bop, sav, smc, extR, extDt, incTx, oilp, inf, excr, capEx have similarities in their posterior means obtained using benchmark prior and proposed prior.

Table 3: Post mean (beta-binomial model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	4.11140	3.86962	4.20166	3.92413
ms	0.12255	0.52684	0.07743	0.35145
gfcf	0.00062	0.00156	0.00046	0.00139
cps	-0.03703	-0.14485	0.00537	-0.05752
recEx	0.39819	0.02487	0.28778	0.15338
bop	-0.00002	-0.00002	-0.00003	-0.00002
sav	0.07572	0.24447	0.11218	0.16869
smc	0.01818	0.01393	0.00500	0.01616
extR	0.00323	0.00137	0.00682	0.00410
extDt	-0.00246	-0.00468	-0.00288	-0.00214
incTx	0.00004	0.00006	0.00011	0.00013
uemp	-1.23118	-1.15740	-0.89399	-0.35026
fd	3.89658	2.68070	4.19600	1.21400
oilp	0.32416	0.06247	0.18981	0.12009
domD	0.10235	-0.00317	0.06019	-0.00345
inf	-0.03542	-0.01078	0.00372	-0.03465
excr	-0.52834	-0.06516	-0.34983	-0.17260
capEx	0.00213	-0.00173	-0.00914	0.03467
lr	-0.07572	-0.10318	-0.03336	-0.02008

Table 3 reveals the similarities in the posterior means obtained using the benchmark prior and the proposed prior for some variables, namely, indQ, gfcf, cps, bop, sav, smc, extR, extDt, incTx, oilp, domD, inf.

Table 4: Post Standard Deviation (uniform model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	2.04060	1.03482	1.88390	1.09424
ms	2.24058	2.04735	2.29010	1.75932
gfcf	0.00397	0.00277	0.00366	0.00286
cps	2.05880	1.54002	1.85790	1.31761
recEx	5.00416	2.43709	4.30139	2.11940
bop	0.00045	0.00029	0.00040	0.00026
sav	3.20830	1.13946	2.26497	1.24570
smc	0.29414	0.10768	0.22005	0.11910
extR	0.09226	0.03975	0.07973	0.03728
extDt	0.15990	0.06644	0.14820	0.07960
incTx	0.00209	0.00080	0.00157	0.00091
uemp	69.04230	27.09680	65.40290	30.77290
fd	178.97790	75.95150	166.23330	86.50998
oilp	2.77580	0.74539	2.63850	0.91367
domD	2.22880	0.43870	2.01700	0.67170
inf	9.66120	2.53540	7.98830	3.05890
excr	14.27200	3.60432	13.97110	4.95790
capEx	2.20898	0.57378	1.70470	0.61780
lr	36.88430	7.04451	29.46310	10.74040

From Table 4 it is seen that there are similarities in the posterior standard deviation of the variables indQ, gfcf, bop, smc, extR, extDt, incTx, capEx obtained using benchmark prior and the proposed prior. However, some variables like, ms, cps, recEx have a smaller posterior standard deviation using the proposed prior. The posterior standard deviation obtained using UIP and HQ, have a lot of similarities, though in many variables HQ gives a smaller posterior standard deviation. The posterior standard deviation obtained from benchmark prior and proposed prior are quite smaller in all the variables than that from UIP and HQ.

Table 5: Post Standard Deviation (binomial model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	2.03740	0.86885	1.83357	0.90412
ms	2.21517	1.91258	1.59955	1.36196
gfcf	0.00408	0.00249	0.00326	0.00277
cps	1.71348	1.43588	1.39634	0.99232
recEx	4.82520	1.62780	4.18781	1.59985
bop	0.00040	0.00023	0.00035	0.00023
sav	2.62066	0.98633	2.09180	1.04074
smc	0.24483	0.11219	0.19781	0.11037
extR	0.09424	0.02746	0.08047	0.03238
extDt	0.15473	0.05819	0.12190	0.07075
incTx	0.00175	0.00069	0.00146	0.00075
uemp	56.44680	22.95131	54.63933	24.36620
fd	152.99600	71.26830	145.25080	55.16940
oilp	2.18950	0.67810	2.02593	0.90272
domD	2.05865	0.50824	1.66414	0.39240
inf	7.94390	2.27279	8.27744	2.17530
excr	12.36748	3.06440	9.34905	3.66540
capEx	2.12900	0.47779	1.63562	0.58720
lr	31.92260	6.97705	26.21276	9.81959

From Table 5, it is observed that there are some similarities in the posterior standard deviation obtained with UIP and HQ, with some variables having a smaller posterior standard deviation in HQ. The posterior standard deviation of the variables indQ, gfcf recEx, bop, sav, smc, extR, extDt, incTx, inf are similar in the benchmark prior and proposed prior. However, some variables, namely, ms, cps, fd, domD have smaller posterior standard deviation in the proposed prior than in the benchmark prior. Furthermore, the proposed prior gave a smaller posterior standard deviation in all the variables than that given by UIP and HQ. The posterior standard deviation of the variables ms and cps are smaller in HQ than in benchmark prior.

Table 6: Post Standard Deviation (beta-binomial model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	1.18534	0.70741	1.12630	0.98481
ms	0.95221	1.15390	0.59296	0.95108
gfcf	0.00249	0.00254	0.00207	0.00268
cps	0.72999	0.80137	0.41145	0.62786
recEx	2.36715	0.34692	1.86270	1.15660
bop	0.00019	0.00013	0.00020	0.00012
sav	0.70549	0.69829	0.66610	0.56337
smc	0.12009	0.07106	0.06167	0.07899
extR	0.02802	0.01211	0.03962	0.02638
extDt	0.06019	0.03560	0.04854	0.02809
incTx	0.00057	0.00047	0.00078	0.00076
uemp	20.78592	11.06576	15.85390	7.05660
fd	65.31914	30.80220	57.71920	54.17030
oilp	1.94932	0.53654	1.50884	0.73340
domD	1.21774	0.08931	1.05360	0.14790
inf	3.60306	0.66705	3.27680	1.42850
excr	6.18971	1.02096	4.75589	1.87490
capEx	0.80126	0.14666	0.60450	0.45410
lr	12.04771	4.01257	5.04290	5.25930

According to Table 6, the posterior standard deviation of the variables gfcf, bop, incTx, smc, extR, extDt, domD obtained from using the proposed prior are similar to those obtained using the benchmark prior. However, the proposed prior gave a smaller posterior standard deviation than the benchmark prior in the following variables: ms, cps, sav, uemp. The posterior standard deviation given by the proposed prior and the benchmark prior in all the variables except ms and cps are smaller than those given by HQ. The HQ produced similar posterior standard deviation with the UIP except in some variables where it gave smaller standard deviation than the UIP.

Table 7: Posterior Inclusion Probability (uniform model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	0.6866	0.9427	0.7704	0.9505
ms	0.2057	0.4627	0.2380	0.3233
gfcf	0.3190	0.4408	0.3165	0.4061
cps	0.2076	0.3056	0.2469	0.2558
recEx	0.3157	0.2237	0.2681	0.1410
bop	0.1728	0.2043	0.1769	0.1548
sav	0.2189	0.1757	0.1916	0.1986
smc	0.2265	0.1373	0.2059	0.1529
extR	0.2730	0.1321	0.2352	0.1228
extDt	0.1765	0.1116	0.1774	0.1194
incTx	0.2763	0.1107	0.1789	0.1366
uemp	0.1583	0.1011	0.1615	0.0997
fd	0.1655	0.1009	0.1638	0.1103

oilp	0.1845	0.0799	0.2102	0.0819
domD	0.1957	0.0689	0.1893	0.0863
inf	0.1444	0.0665	0.1233	0.0642
excr	0.1752	0.0642	0.1856	0.1041
capEx	0.1841	0.0586	0.1261	0.0610
lr	0.1786	0.0457	0.1343	0.0731

From Table 7, the proposed prior shows a higher posterior inclusion probability in 11 variables, namely, indQ, sav, smc, extDt, incTx, fd, oilp, domD, excr, capEx, lr than in benchmark prior.

Table 8: Posterior Inclusion Probability (binomial model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	0.6866	0.9747	0.8010	0.9784
ms	0.2315	0.4849	0.1718	0.3245
gfcf	0.2824	0.3546	0.2161	0.4010
cps	0.1749	0.2736	0.1900	0.1637
recEx	0.3012	0.1316	0.2213	0.0989
bop	0.1460	0.1357	0.1339	0.1235
sav	0.1893	0.1711	0.1522	0.1885
smc	0.1634	0.1140	0.1610	0.1130
extR	0.2557	0.0876	0.2213	0.0936
extDt	0.1730	0.0668	0.1256	0.0977
incTx	0.1979	0.0824	0.1688	0.1020
uemp	0.1161	0.0688	0.1535	0.0787
fd	0.1207	0.0576	0.1476	0.0616
oilp	0.1186	0.0534	0.1336	0.0701
domD	0.1675	0.0665	0.1443	0.0549
inf	0.1034	0.0545	0.1378	0.0378
excr	0.1589	0.0628	0.1236	0.0753
capEx	0.1592	0.0469	0.1252	0.0518
lr	0.1371	0.0435	0.1161	0.0611

From Table 8, the proposed prior shows a higher posterior inclusion probability in 12 variables, namely, indQ, gfcf, sav, extR, extDt, incTx, uemp, fd, oilp, excr, capEx, lr than in benchmark prior.

Table 9: Posterior Inclusion Probability (beta-binomial model prior)

Variable	UIP	BRIC	HQ	proposed
indQ	0.9448	0.9935	0.9500	0.9666
ms	0.0587	0.3027	0.0662	0.2242
gfcf	0.0962	0.3109	0.0671	0.2587
cps	0.0368	0.1079	0.0318	0.0775
recEx	0.0594	0.0105	0.0434	0.0312
bop	0.0330	0.0406	0.0495	0.0305
sav	0.0423	0.1406	0.0690	0.0995
smc	0.0574	0.0482	0.0134	0.0537
extR	0.0382	0.0207	0.0471	0.0363
extDt	0.0318	0.0318	0.0247	0.0189
incTx	0.0273	0.0352	0.0304	0.0473
uemp	0.0158	0.0236	0.0172	0.0121
fd	0.0326	0.0231	0.0312	0.0410
oilp	0.0620	0.0250	0.0441	0.0370
domD	0.0377	0.0044	0.0440	0.0102
inf	0.0224	0.0049	0.0247	0.0157
excr	0.0374	0.0090	0.0440	0.0219
capEx	0.0261	0.0049	0.0179	0.0168
lr	0.0203	0.0121	0.0047	0.0167

From Table 9, the proposed prior shows a higher posterior inclusion probability in 11 variables, namely, recEx, smc, extR, incTx, fd, oilp, domD, inf, excr, capEx, lr than in benchmark prior.

Conclusion

Some of the popular g-priors in literature, namely, uniform information prior, Hannan-Quinn criterion and benchmark prior of Fernandez *et al.* have been compared in the light of the different model priors, which are uniform model prior, binomial model prior and beta-binomial model prior. A new g-prior is proposed and used to compare the existing priors to ascertain its relevance in Bayesian inference. The results in this work show that the proposed prior exhibited dominance over uniform information prior and Hannan-Quinn criterion as it gives smaller posterior standard deviation for all the variables in the analysis. It also showed some similarities with the benchmark prior for most of the variables. However for few variables it gives a smaller posterior standard deviation than the benchmark prior.

We therefore recommend that this proposed g-prior be used in Bayesian model averaging when no prior information about the subject of analysis is available.

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