DEVELOPMENTAL AND INDIVIDUAL DIFFERENCES IN PURE NUMERICAL ESTIMATION

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Abstract

Developmental and individual differences were examined in pure numerical estimation. Children were asked to solve three types of numerical estimation problem: number line, measurement and computational. Children in primary 3, 4, 5 and 6 were presented problems involving the numbers 0-100. Developmental differences involving increasing reliance on the linear representation of numbers and decreasing reliance on logarithmic representations were found across various estimation tasks. This finding agrees with the findings by Siegler & Booth (2006).

Estimation is a universal activity in the lives of both children and adults. Estimation is used to solve many problems because accurate estimates are sufficient for many purposes and because people often lack the knowledge, time, means, or motivation needed to calculate precise values. Despite the importance of estimation in children's lives, not much is known about the process. A large part of the reason for this limited knowledge about estimation is the diversity of tasks that fall under the heading. For instance, estimating the population of Nigeria, the product 186x134 and the speed of a car have little in common except for the answer being approximate. This diversity implies that tasks that fall under the estimation heading will have numerous sources of difficulty and also numerous patterns of development. Research on estimation has presented several difficulties and researchers have had to contend with the various meanings associated to estimation (Rubenstein, 1985).

Developmental and Individual Differences in Estimation

Dowker (2003) found little evidence for consistent developmental or individual differences across different types of estimation when she reviewed the literature on estimation. She found out that while some estimation types showed great improvement in terms of age and experience, others did not. Similarly, existing research on individual differences in estimation is inconclusive (Booth & Siegler, 2006). The few studies that have examined multiple types of estimation have not revealed consistent individual differences across estimation tasks (Dowker, 1998; Hook, 1992, Paul, 1972).

However, previous studies on estimation have shown that more skillful estimators tend to have better conceptual understanding of mathematics (Levine, 1982; Rubenstein, 1985; Petitto, 1990; LeFerve, Greenham & Naheed, 1993), better counting and arithmetic (LeFerve, Greenham & Naheed 1993; Newman & Berger, 1984), greater working memory capacity (Case & Sowder, 1980) and higher mathematics achievement test scores (Siegler and Broth, 2004) than do children who estimate less accurately.

Booth and Siegler (2006) found developmental trend and individual differences in different types of pure numerical estimation regarding elementary school children, hence, the purpose of this study was to replicate Booth and Siegler (2006). However, Booth and Siegler's participants were younger (kindergarteners, first, second and third graders).

Methodology

Participants

Sixty eight children participated: 18 primary 3 (mean age, 9), 17 primary 4 (mean age, 10), 17 primary 5 (mean age, 11) and 16 primary 6 (mean age, 12).The mean ages were to the nearest whole number.

The children were selected from a private school. Participation was voluntary and no extrinsic rewards were provided.

Procedure and Materials

In this study an experiment was performed. Three pure numerical estimation tasks for primary 3, 4, 5 and 6 using different assessment methods for each task were presented. Children were required to estimate (a) the answer to addition problems (computational estimation) (b) the length of a line in inches (measurement estimation) and (c) the location of a number on a line with numerical anchors at each end (number line estimation).

The experimenter met with the children for a 30 minutes session. The session consisted of two phases, one phase involved assessment of number line estimation and the other phase involved exploratory assessments of the other two types of estimation.

The number line phase included an initial orienting problem followed by 26 experimental problems. On the orienting problem, children were presented a sheet of paper with 25cm line across the middle; the number 0 was printed on the left end and the number 100 on the right end. Children were asked to mark where they thought would be the position 50 on the line. After they did so, they were shown an identical number line with 50 marked in the correct position, and were told that that was where 50 belonged, and asked if they knew why 50 was there. The children were told, 'Because 50 is half of 100, it goes directly in the middle, half way between 0 and 100. So 50 is in the middle and it is the only number that goes exactly in the middle.

After the orienting problem, children were presented 26 sheets of paper, each with an identical 25cm line, and asked to put a single mark on each line indicating the position of the number printed on the paper. To ensure discrimination between linear and logarithmic estimation patterns, the numbers below 30 were over sampled by including four numbers from each of the three decades and two numbers from each successive decade. The order of the sheets was randomized separately for each child.

For the measurement estimation task, children were shown a sheet of paper with a 1-inch'line in the middle of the paper. They were told that the line is 1 inch long, and were asked to trace the line with their finger and pay attention to its length. They were also told that the line would remain present to remind them of the length of an inch. This was to eliminate the knowledge of the measurement unit as a source of variability in their estimates. After the 1-inch marker was introduced, children were presented two measurement tasks, one requiring production of length and the other judgment of length. On the production of length task, children were given blank sheets of paper, reminded of the presence and length of the 1-inch marker, and asked to draw lines of 3, 5, 8 and 10 inches. On the judgment of length, children were shown four cards each with a line printed across the middle and two possible measures of its length in inches printed below. They were to choose the measure that was closer to the correct length. The two choices were the line's actual length and a number that was 4 or 5 inches away from the actual length.

For the computational estimation task, children were shown 12 cards, each containing an addition problem, and three possible answers to the problem. All of the sums were between 0 and 100. The alternative answers listed for each problem were always consecutive multiples of 10. Each problem was read aloud by the experimenter, and the children were asked which of the three choices was closest to the answer (e.g. Is 35 + 23 closest to 50, 60 or 70). There was no time limit for any of the problems.

Results

Number Line Estimation

To measure changes in estimation accuracy, each child's percent absolute error was calculated thus:

$$\frac{\text{Estimate} - \text{Estimated Quantity}}{\text{Scale of Estimates}}$$

For instance, if a child was asked to estimate the location of 24 on a 0-100 number line and placed the mark at the location that corresponded to 39, the percent absolute error would be 15%:

$$\left[(39 - 24) \Big/ 100\right]$$

An analysis of variance (ANOVA) on each child's mean percent absolute error was conducted. The ANOVA indicated that accuracy increased with class or grade, F(3, 65) = 13.26, p-0.05. Primary three's mean percent absolute error (21%) was greater than those of primary four, five and six (19%, 12% and 8%, respectively). The summary of the analysis of variance (ANOVA) is given in Table 1.

Table 1: Summary of ANOVA **for the Number Line Estimation**

Source	**DF**	SS	**MS**	**F**	P
Factor	3	1899.9	633.3	13.26	0.000
Error	65	3103.3	47.7		
Total	68	5003.2			

For the measurement estimation task, performance was evaluated for the two tasks. Skill at length production task was measured by percent absolute error on the drawing of the lines. A one-way analysis of variance (ANOVA) was conducted to find out changes with age and grade. The result of the test showed significant difference with respect to class and age, F(3, 65) =24.05, p<0.05. The percent absolute error of the lines of primary 6(0.67%) was less than those of primary 5(1.08%), primary 4(3.4%) and primary 3(3.2%).

However, the results showed that the percent absolute error of primary 4 was greater than that of primary 3. Table 2 shows the summary of the ANOVA for length production.

Table 2: Summary of ANOVA for Length Production

Source	**DF**	SS	MS	F	P
Factor	3	102.20	34.07	24.05	0.000
Error	65	92.06	1.42		
Total	68	194.26			

On the length judgment, proficiency was measured by the percentage of problems on which the pupils chose the better estimate. Proficiency increased with class or grade and age except between primary four and three, F (3, 65) =12.35, p<0.05. The proficiency of primary six (87.5%) was greater than those of primary five (51.51%) primary four (39.74%) and primary three (46.07%). The proficiency of primary three (46.07% was greater than that of primary four (39.74%). The summary of the ANOVA for the length judgment is given in Table 3.

Table 3: Summary of ANOVA for the Length Judgment

Source	**DF**	SS	MS	F	P
Factor	3	22.564	7521	12.35	0.000
Error	65	39585	609		
Total	68	62149			

On the computational estimation, performance was measured by the percentage of problems on which the pupils selected the more accurate estimate. Primary six pupils (65.63%) were more accurate than primary five pupils (51.47%), and both were more accurate than primary four and primary three pupils (42.12% and 40.38% respectively), F(3,65)=7.68, p<0.05.The summary of the ANOVA is presented in

Table 4: Summary of ANOVA for Computational Estimation

Source	DF	SS	MS	F	P
Factor	3	6733	2244	7.68	0.000
Error	65	18996	292		
Total	68	25729			

Discussion

The results of this experiment showed a shift from reliance on logarithmic representation of numerical magnitudes to reliance on a linear representation of them. According to Booth and Siegler (2006), reliance on a logarithmic representation would increase the difficulty of learning the magnitudes of answers. Estimation accuracy increased with age and class. The percent absolute error of primary six pupils on the number line estimation was less than half those of primary three and four pupils. The substantial improvement in estimation accuracy during this period agrees with Booth and Siegler (2006). Similarly, on the length production task, the result indicated improvement in accuracy with class and age. The percent absolute error of primary six pupils (0.67%) was less than half those of primary four and three pupils. This is an evidence of a shift from logarithmic representation (o linear representation. On the length judgment task, there was also evidence of increased proficiency with respect to class and age.

On computational estimation (ask, performance improved substantially in this age period. Primary six pupils were more proficient in this task than primary five, and both were more proficient than primary four and primary three pupils. This finding agrees with the finding by Booth and Siegler (2006)"

Conclusion

This experiment has indicated developmental and individual differences in estimation. The performance on each task improved with class and age. The deviance in measurement estimation task, (where primary four pupils were less in proficiency on length judgment than primary three pupils a.nd less in accuracy on length production than primary three) could probably be because of the few number of items. Four items each for length production and length judgment were used. The tasks used in this experiment were inadequate for answering the questions about the strength of relations among the different types of estimation because of their limited number of items and extremely variable response formats.

However, as noted earlier in the discussion about the danger of relying on logarithmic representation, it is recommended that teachers should present spatial displays of the linear magnitudes associated with operands and answers during learning. This might hasten the learning, of correct answers and as well help children generate plausible rather than implausible errors.

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