MATHEMATICAL MODELS OF VISCOELAS BEHAVIOUR OF PLASTIC MELT IN AN EXTRUDING MACHINE

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Abstract

This paper examines the viscoclastic behaviors of plastic melt during extrusion processes in an extruding machine. These viscoelastic behaviors are stimulated by using simple physical models which are analogous to discrete molecular structure. By so doing, the response of the plastic materials during the extrusion processes is perfectly understood. Some of the models discussed in this paper arc the Maxwell, the Kelving and the Voigt models. This paper also makes use of the principle of superimposition by superimposing Maxwell and Kelvin models to form Maxwell --Kelvin model similarly Kelvin-Voigt model was designed. Deformation equations are introduced to predict creep, relaxation and recovery characteristics of each model. In each ease, the governing equation is established which forms the basis for analyzing the model response—At the end of the day, the designed superimposition Maxwell-Kelvin results is compared to a standard linear solid model. The comparism shows a close correlation and this, of course, confirms the validity of the finds of this paper.

Introduction

Viscoelastic is a material property as a result of an hybrid between perfectly elastic (Kookean) solid and perfectly Viscous (Newtonian) fluid. The fact that thermoplastics at room temperature behave in a similar fashion to metals at high temperatures therefore, the design procedures for relatively ordinary load-bearing applications must always take into account the viscoelaslic behaviors of plastic. (Crawford, 1987)

The main objective of this paper therefore, is to model and simulate the Viscoelastic behaviors of plastic melt during an extrusion process using Maxwell, Kelvin and Voigt approach.

A good result was obtained by superimposing Maxwell and Kelvin models to form what is being called Maxwell-Kelvin model. Similar model-called Kelvin-Voigt model was equally designed and the two models compared .

The result shows that the most characteristic features of viscoelastic materials are that they exhibit a time dependant strain response to a constant stress (creep) and a time dependant stress response to a constant strain relaxation (Duv-devani, 2006). In addition when the applied stress is removed the materials have the ability to recover slowly over a period of time (Charles, 1992). Both plastics and metals displace these characteristics, while the former absence them at room temperature the latter at high temperatures. (Ojo, 2005). Simulation of the Mathematical Models. Model 1.

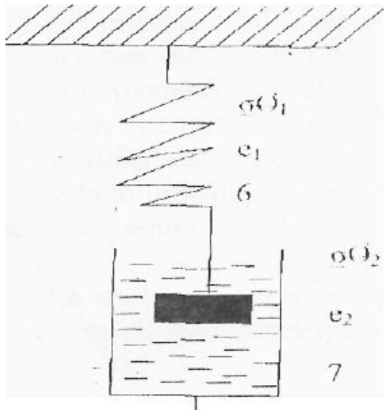


Fig 1. The Maxwell model. (Learn, 2004)

This model consists a spring and dashpot in series as shown in fig 1 above.

(a) **Deformation Representation**

fin, = 6e, for the spring (Tyler, 2002} ---------------------------------- (1)

n< j, = 7e, for the dashpot (Tyler, 2002) -------------------------- - (2)

For equilibrium of forces

,-, (> = ,,, j 1 = n<)2 (Ryder, 1987) --------------------------------- --(3)

(assume constant area). Where n O- applied stress

e = e, + e_2 (Ryder, 19H7)-------------------------------------- ---(4)

Where

e = total strain

from equations (1), (2) and (4) we have:-

e = 1/6 n(i, + 1/7 ---;! $)_3$ -------------------------------------- ....(5a)

or

e- .< V& + ni $)_2$/7 —(5b)

Equation (5b) is the governing equation for Maxwell model. —

(b) **Creep Representation**

Assume constant stress, equation (5b) bccomes:-e = n< i/6 + < . (V7

:- e = e = n(i (6 + *II* 67)-- —(6)

equation (6) indicates a constant rale of increase of strain with time Thus

e (t) = r,(\ (6 + *II* 67).t -- (7)

and creep modulus

E (I) = ni Ve(t) = (67/6 + 7).t -........ --(8)

(c) **Relaxation Representation**

Assume constant strain; equation (5b) becomes:-

O= I / 6 n (i , + l/7crU$_2$

i.e. 7 o ( ) $_1$ + 6n0$_2$/87 = 0

7 r > (> i + 6a()$_2$ =O. for *r,(*)| = <-,*(*)$_2$ = n(i

equilibrium then

(7 + 6)aU = O. for initial condition nt j = n(i_0, at t = to; solving differential equation yields

nO(E) =c)0_0 exp(7 + 6)t - ...... ---------------------------- (9a)

Equation indicates that the stress grows exponentially with time constant (7+8)

(d) **Recovering Representation**

when the stress is removed, equation (5b) becomes

e = 1/7 + 1/6 i.e. e(t) = exp (7 + 6 / 7 + 6)------- (9b)

this is an elastic recovery.

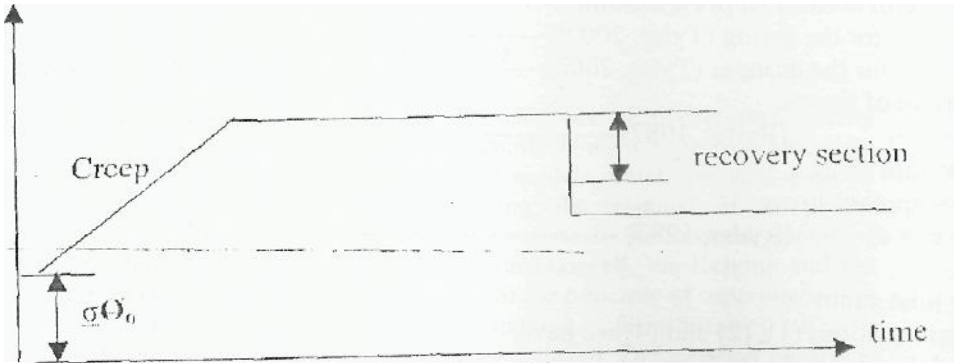
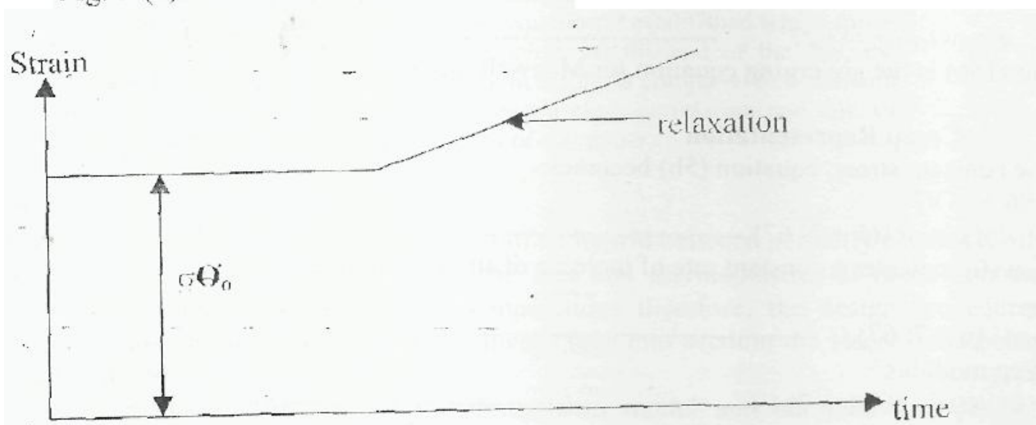


Fig. 2 (a) Relaxation of Maxwell model



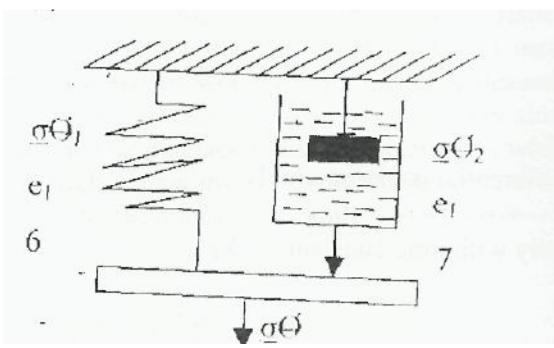


Fig 3. The Kelvin model (Learn, 2004)
This model is a spring and dashpot connected in parallel
(a) Deformation representations
n<-i = r,(>! + o( )2.. - (10)
p = p. = (*„ (11)
c C| e2……………………………………………………………….
from equation (11)
n( ) = fip + 7p (12)

Equation (12) is the governing equation for Kelvin model.
(b) Creep representation
"Assume constant stress" equation(12) can be solved differentially to get
e(t) = g(V6 [1 -exp(6/7)t] -(13)
equation (13) indicates an exponential increase in strain from 0 to 6/7.

(c) Relaxation representation

Assume constant strain equation (12) becomes

(141 \i<+)

Solve differently

'"i <)(t) = e[exp (6 + 7)0 ---------------------------------- —(15)

There is a exponential growth in stress from O to (6 = 7).

(d) Recovery representation

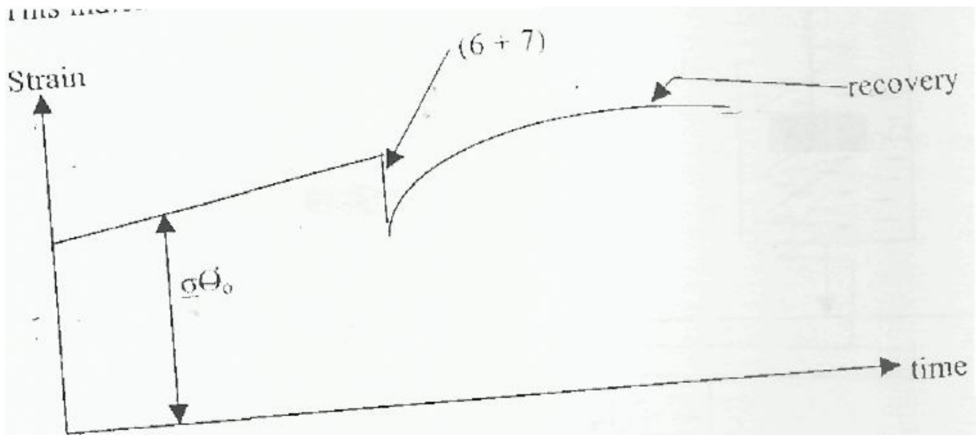
When the stress is removed equation (12) becomes

o=

Solving differentially (Stroud, 2003)

e(t) = e[exp (6 + 7)] -(17)

This indicates an exponential growth in strain from O to (6 + 7).



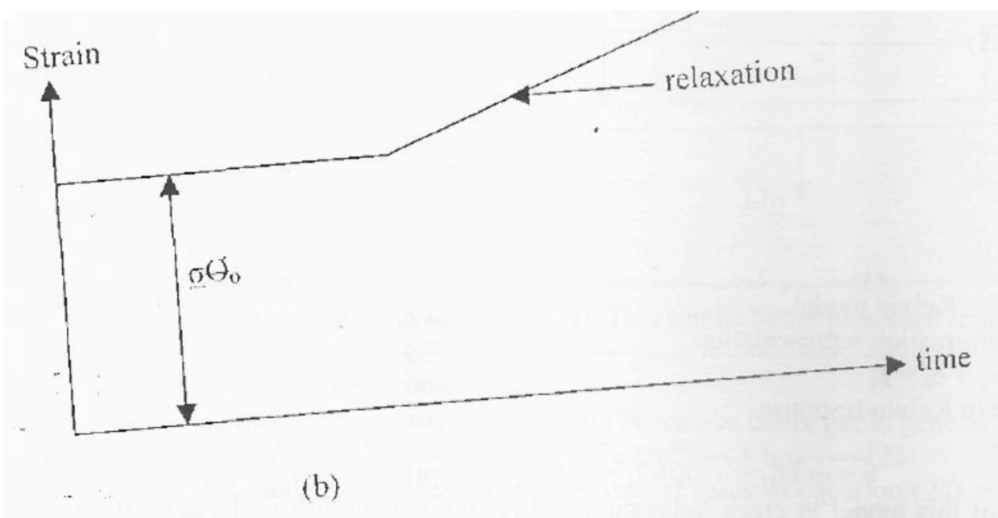


Fig 3. Response of modified Kelvin model.

Model 3

This model exploys the principle of superposition. Whereby the Maxwell model is superimposed on Kelvin model to form what we called Maxwell - Kelvin model.

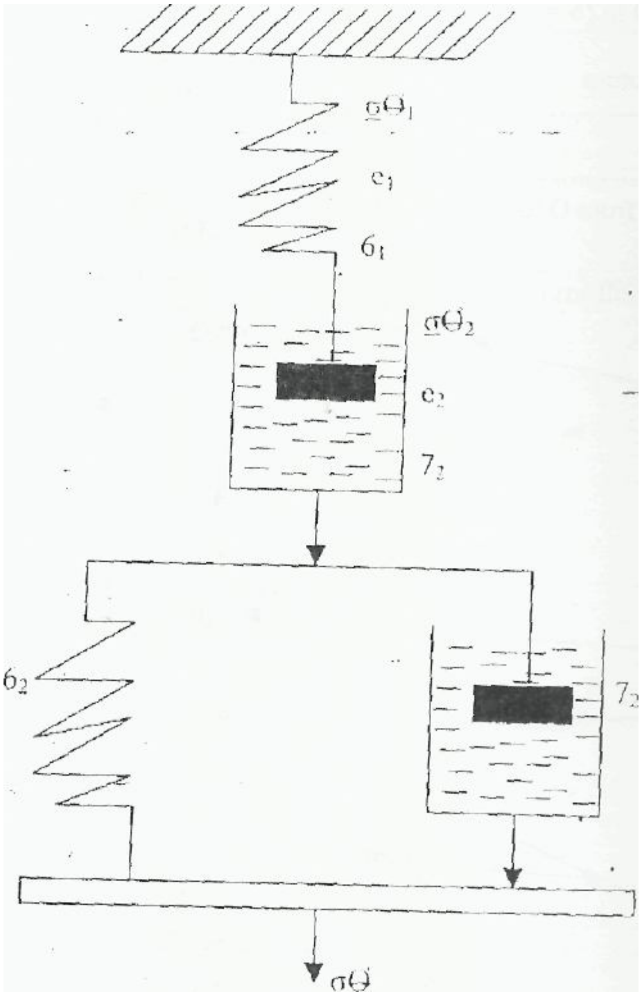


Fig 4, Maxwell - Kelvin model.

(a) Deformation representation

Total strain = e, + e_2 + e_k

where Ck is due to Kelvin response.

Strain rate is :-

e - <-< i/6, + *a* «)(t)/7, + a< >/6_2 [1 - exp(6_2/7_2)t] ~ ----- (18)

The response of this model to creep, relaxation and recovery situations is the sum of the affects of both Maxwell and Kelvin models such that For creep:

e(t) = fT(i (6 + 7 / 67)t + r>(i/6 [1 - exp (6/7)t| - p(
)/6(t) f (M- 6/7) -l- 1 - exp (6/7)]

i.e. e(t) = cr( )/6(t) [(2 + 6/7) - exp (6/7)] …………--- (19)

for relaxation

*n (*J(t) = a( )$_0$exp (7 -I 6)1 + c[cxp(6 f- 7)1 J

i.e. " _r> ()(t) = (o(-)$_0$"+ e)cxp(7 4 6)t ……………………. ----------(20)

for recovery

e(t) = exp(7/6 / 76) + c[exp(6 -\ *1}*\

i.e. e(t) - [e - exp(l/76)j exp (7 -f 6) ------ ...... ------ ....

Equation {18) is the governing equation for model 3 which is based from the superposition principle. The coefficients for the three right hand terms are just the expressions of material constants which are: , 1/7, and 1/62 [1 exp(():/7:Hj

Judgment of the Results

The validation of the results can be made3 by comparing it with a standard linear solid. This is a model consisting of elements in series and parallel as shown in fig 5.

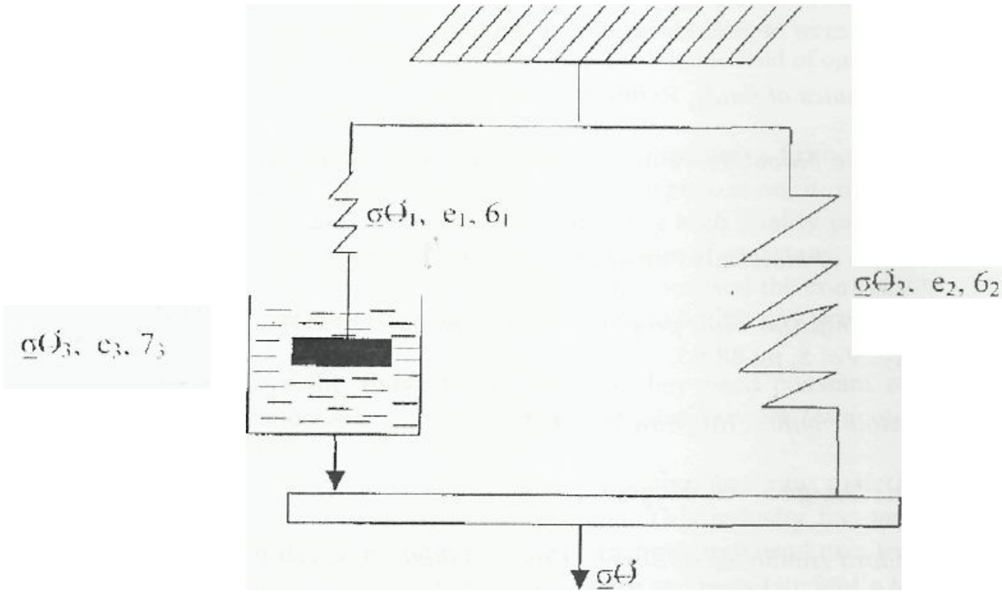


Fig 5. Standard linear solid model. for equilibrium
n(j

(22)

)$_3$, *c>(> - n(*), + qO_2 (Ryder 1987)

Deformation representation

e = e_2 = C| + e_3

from equation (22) n<), = n(i - o( )$_2$

and nt)$_3$ = ni i - *n(*)$_2$

e- n(i- 6_2 e/ 6, + <-;( ) - 6_2e/7,

rearranging to get

7_3a + 6ia = 7_3 (6 1 + 6_2)e + 6,6,e ------------------------------ —(23)

Equation (23) is the governing equation for this model. This can be better presented in the form

a,a + a,,o = b,e + b_0e ………………………………………………… (24)

where a i , a,,, b] and b_0 are al I material constants.

In the more modern theories of viscoelasticity equation (24) can be better put in general form thus:-

a_n 6" a / $6t^n$ + a_n . i $6^{n\ "\ !}$ / $6t^n$ + — + $a_($,o - b_m 6^me / $6t^m$ — + b_ne -- -(25)

It should be noticed that models described earlier are special cases of equation (25) valid for design purposes.

Conclusions

-This paper has presented some modified viscoelastic models, analyzed them and superimposed them to obtain a new model. The response of this model in terms of creep, relaxation and recovery agreed well with that of a standard linear solid model which acts as validation to the new model. The paper has thus, opened wide options for more models to be developed. Also a standard basis is get for the purpose of comparism and justification.

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