SOME APPLICATIONS OF MATHEMATICS TO ECONOMICS

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Abstract

Most problems that occur in social and managerial sciences often require (he use of mathematics for their solutions. Some of these aspects of mathematics that are used to get the solutions of these problems include matrix algebra, mathematical modeling, differential equation and a host of others. In this paper work, some economics problems are being solved using simple mathematical techniques and we have also illustrated how some concepts in economics are transformed into mathematical form. This is not to say that only simple mathematics is involved in the solution of economics problems. There are also some complicated mathematics derived from the study of economics problem, but in this work we will be concerned solely on the use of simple mathematical methods only.

Introduction

Historical Background of Mathematics in Economics

Mathematics first took on a significant role in economics in the last century in the building of what is commonly referred to as the "Marginalist Revolution". This was a period in which classical concern with production, growth and distribution of the fruit among social classes were being replaced with market exchange. The focus thus shifted from the level of economy and the social classes to the level of individual. As a result of this shift in focus, Augustin Cournot then introduced the systematic application of mathematics to economy. Apart from Cournot, many famous mathematicians and even economists have now seen the application of mathematics in economics because by this stage, mathematics has taken over from the physics as a model. Macroeconomics has emerged as a mathematical system quite separate from the microeconomics. Modern economics thus relies heavily on mathematics.

Decision makers (e.g. consumers, firms, government) in standard economic theory are assumed to be "rational". That is each decision - maker is ordering over the outcomes to which her action, among those feasible that is most assumed to have a preference actions lead and to choose an preferred.

Literature Review

Mathematics plays a very important role in economics. This role has been significant for almost a century, and has been increasing in important particularly in recent years. Most problems that occur in social managerial sciences often require the use of mathematics for their solution. Mathematical economics refers to the application of mathematical methods to represent economic theory or analyze problems, posed in economics. Much of modern economics can be presented in geometric terms or elementary mathematical notation. Mathematical economics however refers conventionally to the use of methods such as calculus and matrix algebra in economics analysis. These are pre - requisites for formal study, not in mathematical economy alone but also in contemporary economic theory generally. Some aspects of mathematics that are used for solving these problem include real analysis, matrix algebra, mathematical modeling, differential equation and a host of others (Ego, 1996).

Mathematics is increasingly important in terms of expression and communication of ideas in economics. This applies at a variety of levels; from school people making subject choices to policy makers' understanding of policy advice (Dow, 1999).An economic problem often involves so many variables that mathematics is the only practical way of "handling it" - handling in the sense of solving it.

Economics analysis relies more and more on mathematical foundations. Economics has become increasingly dependent on mathematical methods and the mathematical tools it employs have become more sophisticated. One's interpretation is that economics has been undergoing technical change, employing more mathematics and more sophisticated statistical techniques, which have improved the productivity of the discipline. The change in content is thus one of the undoubted improvements (Dow, 1999).

Applied mathematicians apply mathematical principles to practical problems, such as economic analysis and other economics related issues in such a way that many economic problems are often defined as being integrated into the scope of applied mathematics.

An Economy Model

Matrix algebra has proved effective in analyzing problems concerning the input and output of an economic system. Suppose an economic system has in different industries, say; I,, 12, la,.... I_n, each of which has input needs (raw material, utilities) and an output (finished products). The input coefficient dij measures the amount of input the $j^{'1}$ industry needs from the i^{th} industry to produce one unit. The collection of input coefficient is given by the following n x n matrix

$$\text{Users: } I_1, \quad I_2, \ldots \quad I_n$$

$$D = \begin{bmatrix} D_{11}, & D_{12}, \ldots & D_{1n}, \\ D_{21}, & D_{22}, \ldots & D_{2n}, \\ . & . & . \\ . & . & . \\ . & . & . \\ d_{n1}, & d_{n2}, \ldots & d_{nn}, \end{bmatrix} \begin{matrix} I_1 \\ I_2 \\ . \\ . \\ . \\ I_n \end{matrix} \Bigg\} \text{ Suppliers}$$

This matrix is called the input-output matrix.

To understand how to use this matrix, imagine that the entries of D are given dollars. The total amount spent by the j^{lh} industry to produce one dollar worth of output is given by the sum of the entries in the j^{th} column, Thus, for a matrix to qualify as an input - output matrix, it must have the following characteristics;

a) The matrix must be square, of order n x n.
b) Each entry in the matrix must be non - negative and less than or equal to 1. That is;0= < dy <
1.
c) The sum of the entries of any column must be less than or equal to 1. That is,

d) DU- + DZJ + D_{3j}+...+d_{nj}< 1

e) **Closed and Open Economic** System

f) If the economic system is closed (meaning that it sells its products only to industries within the system), then the total output of the i^{th} industry is given by the linear equation; X = dj,Xi + dj2X2 + ... + $d_{ia}X_n$

g) Thus, to satisfy the demand for its product the i^{th} industry must produce d^x, units for the first industry, $d_{i2}x_2$ units for the second industry, and so on. The entire system of the equation for a closed model is as follows;

h) $X_1 = d_{11}X_1 + d_{12}X_2 + \ldots + d_{1n}X_n$

i) $X_2 = d_{21}X_1 + d_{22}X_2 + \ldots + d_{2n}X_n$

j) $X_n = d_{n1}X_1 + d_{n2}X_2 + \ldots + d_{nn}X_n$

k) For a closed economic system, the sum of the entries in each column must be precisely 1. That is;

l) $d_{1j}+d_{2j}+d_{3j}+\ldots$
$+d_{nj}=1$ The matrix form of this system is

m) For an open system (meaning that it sells its product to groups outside the system), the total output of the i^{th} industry is given by;

n) $X_i = d_{i1}X_1 + d_{i2}X_2 + \ldots + d_{in}X_n$

o) Where e_i represents the external demand for the ith industry's products. The collection of

p) total outputs for an open system is therefore represented by the following system of n linear

q) equation;

r) $X_1 = d_{11}X_1 + d_{12}X_2 + \ldots + d_{1n}X_n + e_1$

s) $X_2 = d_{21}X_1 + d_{22}X_2 + \ldots + d_{2n}X_n + e_2$

t) $X_n = d_{n1}X_1 + d_{n2}X_2 + \ldots + d_{nn}X_n + e_n$ The matrix form of this system is X= DX+E Where X is the output matrix and E is the external demand matrix.

u) Example:

v) A system composed of three industries with the following output:

1) To produce one dollar worth of output, industry A requires the following:

w) $ 0.10 of industry A's product

x) $ 0,15 of industry B's product

y) $ 0.23 of industry C's product.

2) To provide the dollars worth of output industry B requires the following:
$ 0.43 of industry A's product
$ 0.00 of industry B's product
$ 0.03 of industry C's product.

3) To produce one dollar worth of output, industry C requires the following.
$ 0.00 of industry A's product
$ 0.37 of industry B's product
$0.02 of industry C's product
Thus the input - output matrix for this system is as follows: User

Users

A	B	C	
0.10	0.43	0.00	A
0.15	0.00	0.37	B
0.23	0.03	0.02	C

Supplier

Finding the Output Matrix of an Open Economic System.

Using the above example to solve for the output matrix X in the equation X = Dx + E, where the external demand is given by

$$E = \begin{pmatrix} 20{,}000 \\ 30{,}000 \\ 25{,}000 \end{pmatrix}$$

Solution
Letting I to be the identity matrix, we can write the equation
X= DX+E as
IX = DX + E, which means that
IX-DX=E or(I-D)X=E
Using the matrix D found in example 1 produces

$$I = D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.10 & 0.43 & 0.00 \\ 0.15 & 0.00 & 0.37 \\ 0.23 & 0.03 & 0.02 \end{pmatrix}$$

$$= \begin{pmatrix} 0.90 & -0.43 & 0.00 \\ -0.15 & 1.00 & -0.37 \\ -0.23 & -0.23 & 0.98 \end{pmatrix}$$

Then, applying Gauss-Jordan elimination to the system of linear equation represented by

(I-D)X= E produces

$$\begin{pmatrix} 0.90 & -0.43 & 0.00 & 20{,}000 \\ 0.15\ 1.00 & -0.37 & 30{,}000 & \\ -0.23 - 0.03 & 0.98 & 25{,}000 & \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 48{,}616 \\ 0 & 1 & 0 & 51{,}058 \\ 0 & 0 & 1 & 38{,}014 \end{pmatrix}$$

Therefore, the output matrix is:

$$\begin{bmatrix} 46{,}616 & A \\ 51{,}058 & B \\ 38{,}014 & C \end{bmatrix}$$

And we conclude that the output for the three industries is as follows:

Output for industry A: 46, 616 units Output for industry B: 51, 058 units Output for industry C: 38, 014 units

Solving for the Output Matrix of a Closed Economic System

A closed economics is made up of four different industries, each of which uses product from the other three industries, as shown below;

Supplier		Users		
	Industry 1	Industry 2	Industry 3	Industry 4
Industry 1	0.15	0.25	0.35	0.05
Industry 2	0.30	0.15	0.20	0.40
Industry 3	0.35	0.40	0.30	0.25
Industry 4	0.20	0.20	0.15	0.30

If industry 4 produces 17,730 units in a year, How many units do the other three industries produce?

Soln:

Observe that the sum of each column in the above matrix is 1, which satisfies the condition for the entries in a closed economic system. Thus, the input - output matrix is a legitimate model for a closed economic system. The system of linear equation representing this system is as follows:

Xi = 0.1Sxi +0.25x2+ 0.35x_3 + 0.05x_4 X_2 = 0.30x, + 0.15x_2 + 0.20x_3 + 0.40x_4 X_3 = 0.30X! + 0.40x_2 + 0.30x_3 + 0.25x_4 X_4 = 0.20x; + 0.20x_2+ 0.15x_3 + 0.30x_4

In standard form (i.e I – X)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.15 & 0.25 & 0.35 & 0.05 \\ 0.30 & 0.15 & 0.20 & 0.40 \\ 0.35 & 0.40 & 0.30 & 0.25 \\ 0.20 & 0.20 & 0.15 & 0.30 \end{pmatrix}$$

This system becomes,

$$0.85x_1 - 0.25x_2 - 0.35x_3 - 0.05x_4 = 0$$
$$-0.30x_1 + 0.85x_2 - 0.20x_3 - 0.40x_4 = 0$$
$$-0.35x_1 - 0.40x_2 + 0.70x_3 - 0.25x_4 = 0$$
$$-0.20x_1 - 0.20x_2 - 0.15x_3 + 0.70x_4 = 0$$

Finally, using Gauss - Jordan elimination, we obtain

X-i 1909 X_4, X_2 = 2 i 74 X_4, and X. = 2830
1773 1773 1773

Since X, = 17,730, the four industries must produce the following numbers of units.

Output for industry 1 = 19,090 units

Output for industry 2 = 21,740 units Output for industry 3 = 28,300 units Output for industry 4 = 17,730 units

Marginal Concepts

The marginal quantity of any function can be equivalently defined as the derivatives of that function.

To show this let us take any function say; C = F (x),

Where C = total cost

x = output.

Marginal cost in economics is defined as the change in total cost incurred from the production of an additional unit. Marginal revenue is defined as the change in total revenue brought about by the sale of an extra good. In order words marginal cost is the rate of change of total cost when output is increased by one unit i.e. marginal cost;

(MC) = F(x+l)-P(x);

But

$$F(x+l)-F(x) = \frac{F\ fx + 1) - F(x')}{f_x + 1) - X}$$

Since Total cost (TC) and the Total revenue (TR) are both functions of the level of output -.: (Q). Marginal cost (MC) and Marginal revenue (MR) can be expressed mathematically as derivatives of their respective total functions.

Thus if TC = TC(Q); then

MC = and

If TR = TR(Q), then $\frac{dIR}{dQ}$

MR=

Form the above; we can see that that marginal concept in any economic function can be expressed as the derivative of its total function.

Example:

If $TR=75Q-4Q^2$'then:

$$MR = \frac{dTR}{dQ} - 75 - 8Q$$

If $TC = Q^2 + 7Q + 23$, then;

$MC = dTC = 2Q + 7$ dQ

Optimizing Economic Function

The economist is frequently called upon to help a firm maximize profits; levels of physical output and productivity, as well as to minimize costs, levels of pollution and the use of scare natural resources. This also can be achieved using the techniques developed earlier; that is the marginal cost and marginal revenues.

Example:

Maximize profits Π n for a firm, given total revenue

$R - 4000Q - 33Q^2$ and total cost $C = 2Q^3 - 3Q^2 + 400Q + 5000$, assuming $Q>0$.

Soln:

a) Set up the profit ji = R -C

Π $JI = 4000Q - 33Q^2 - (2Q^3 - 3Q^2 = 400Q - 5000)$

$= 4000Q - 33Q^2 - 2Q^3 + 3Q^2 + 400Q - 5000$

$= - 2Q^3 - 30Q^2 + 3600Q - 5000$

Conclusion

It can therefore be seen from the various example that mathematical methods play a very vital role in solving most economic problems. This however has made the course mathematical economics of great interest to not only mathematicians but economist alike.

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