ESTIMATION OF LINEAR DISTRIBUTED LAG (KOYCK) MODEL HEAVILY TROUBLED WITH AUTOCORRELATION

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Abstract

This study examined a specification of independent variable often associated with linear autoregressive distributed Lag model called the Koyck model when it is heavily troubled with autocorrelation. Focus is on the Ordinary Least Square (OLS) method when correlated for AR, MA, ARMA and Two Stage Least Square (2SLS) method, when corrected for AR, MA, ARMA. The specification of the independent variables considered is such whose data are generated so that they resemble data obtained from controlled laboratory experiment. The primary emphasis of this study is examining the simple properties of OLS, OLS-AR, OLS-MA, OLS-ARMA and 2SLS, 2SLS-AR, 2SLS-MA, 2SLS-ARMA with a sample size of n = 20, 40 and $\rho = 0.4$, 0.6 and 0.8. Monte-Carlo experiment is employed here as a means of empirical examination. It is found that OLS dominates when the sample size is small and $\rho = 0.4$ and for increased sample size and small ρ OLS, OLS-AR, OLS-ARMA compete favourably and when sample size is increased and for high ρ OLS-AR dominates.

When autocorrelation is present, the OLS estimates although linear, unbiased and asymptotically normally distributed are no longer BLUE and as a result, the usual t, F and x 2 test may not be valid (Damoder, 2006). Although econometricians have paid much attention in recent time to distributed Lag model, the behavioural mechanization of Linear estimators in the Koyck model are not well documented, (Adiele and Nwabueze, 2010), which means that when the estimators are adjusted to filter autocorrelation in a heavily autocorrelated model, the parameters may be Blue.

The Linear model with auto-correlated error terms has found widespread applications especially in modeling economic data. In regression analysis, the usual assumption of independent error term may not be plausible in most cases, especially when using time series data on a number of micro-economic units. This is because, in time series data, the stochastic disturbance terms in part reflect variables which are not included explicitly in the model, which may change slowly overtime or as a result of distributed Lag which may be geometric.

The geometric distributed Lag model, after application of the so called Koyck transformation is often used to establish the dynamic link and estimation in the time series data.

Consider the classical statistical linear model

$$Y = X\beta + U \qquad (1.1)$$

which explains the relationship between a dependent variable *Y*, *K* – 1 explanatory variables *X*, and disturbance term *U* for a sample of *T* observations on *Y* and the *X* „s, where *Y* is a (*T* x 1) vector of observations on a sample space defined on the Euclidean line, *X* is a known (*T* x *K*) nonstochastic design matrix of rank *K*, *β* is a *K*-dimensional fixed vector of unknown parameters, and *U* is a (*T* x 1) vector of unobservable random variables with mean *E* (*U*) = 0 and a finite covariance matrix.

The usual assumption(s) of many data-generating processes is that the elements of the random vector *U* are identically and independently distributed in which case, the covariance matrix takes the form *E* (*UU* „) = σ2*I T* , where, the scalar σ2 is unknown and *I T* is a *T th* –order identity matrix, a more general formulation of the covariance matrix is represented by $\sigma^2\psi = \phi$, where ϕ. is a known positive definite matrix.

A major basic assumption in the classical linear regression model is that the disturbance covariances at all possible pairs of observation points are zero. We refer to the situation as lack of autocorrelation or serial correlation of the error terms. But if the value of the disturbance term *U* in any particular period is correlated with any other value of *U* in the series, we have autocorrelation or serial correlation of the random variable *U*. In regression analysis if the regression model includes not only the current but also the lagged values of the explanatory variables (x"s), it is called a distributed Lag Model while if it includes the Lagged values of the dependent variable among its explanatory variables, it is called an autoregressive model, dynamic or koyck model when transformed.

Many models with autocorrelated error terms have been discussed in the literature. These range from the early works of Anderson (1942), Cochrane and Orcutt (1949), Durbin and Watson (1950, 1951), Hannan (1957), to the recent works of (Podder, and Khatrachyan, 2006. This work will make way for correct inferences to be made in autoregressive distributed Lag Koyck linear model in an effort to do this we resort to Monte Carlo Experimentation undertaken in this study.

When autocorrelation is present, the OLS estimators although linear, unbiased and asymptomatically normally distributed are no longer BLUE and as a result, the usual t, F and x 2 test may not be valid Damoder,(2006). Although econometric has much attention in recent time to distributed Lag mode, the behavioural mechanization of the linear estimators in the Koyck model are not well documented (Adiele,and Nwabueze, 2010), which means when the estimators are adjusted to filter autocorrelation in a heavily autocorrelated model, the parameters may be BLUE.

Scope and Objective of the Study

It is a well-known fact that in the distributed Lag (Koyck) Model, as well as the adaptive expectations model and several other models, the stochastic explanatory variable Y_{t-1} is correlated with the error term V_t. If an explanatory variable in a regression model is correlated with the stochastic disturbance term, then some estimations are not only biased but also inconsistent, that is even if the sample size is increased indefinitely, the estimators do not approximate the true population values.

However, most of the existing estimation methods have some desirable properties; but the degree of the correlation might affect these properties. The various method of parameter estimation in Linear Models with autocorrelated error terms have known asymptotic properties, while their sampling properties are yet to be well investigated and understood, Gujarati (2006). Furthermore, the behaviour of different degrees of correlation on the estimation methods needs to be well investigated and understood. Since Monte-Carlo experiments provide means of modeling small sample properties of estimators, we would resort to it in order to study these properties.

In this study, attention will be focused on carrying out a monte-carlo experiment on the Koyck model with different estimation methods, comparing the estimation methods in order to rank their performances in the presence of first order auto correlated disturbance terms in Koyck Models. Examine the effects of the degree of autocorrelation of the disturbance terms on the different methods of estimations and investigate the effects of different sample sizes and replications of the experiment on the different methods of estimations.

Review of Previous Monte Carlo Studies on Autocorrelation

The efforts of many researchers in this direction have been very rewarding. Cochrane and Orcutt (1949) brought to focus the attention of economists to the fact that the presence of auto correlated error terms requires some modifications of the usual ordinary least square method of estimation. They also recognize the fact that many current formulations of economic relations are highly positively auto correlated.

Koyek Model

For estimating the Koyck model, OLS is consistent though biased. Instrumental variable estimation is possible (Maddala-Kim, 1998). According to Durbin (1960) estimating functions viewpoint, OLS is optimal despite lagged dependent variable. But if addition to *y t-1* on the right side, there is a problem of auto correlated errors, then they too can be handled by Durbin 1960 two-step estimator described in Gujarati (1995:432). Somehow this correlation can be removed and then one can apply OLS to obtain consistent and unbiased estimates as have been shown (Adiele and Nwabueze, 2010). To this, Liviation (1963) proposed finding a proxy for ***Y t-1*** that is highly correlated with ***Y t-1*** but uncorrelated with *V t, where V t* is white noise. Such a proxy is called instrumental variable. Liviation advocated for *X t-1* as the proxy, yet *X t-1*, and *X t* could be correlated which also exposes this approach to multicollinearity.

The Monte-Carlo Approach

The Monte Carlo approach is the process of inferring conclusions from simulated data. In the theoretical or analytical approach, conclusions are deduced from postulates. Basically, the difference is between induction and deduction. In econometrics, while asymptotic properties of estimators obtained by various econometric techniques are deduced from postulates or self-evident assumptions, an approach that is often described as analytical, small sample properties of such estimators have always been studied from simulated data in what are known as Monte Carlo studies. The use of this approach is due to the fact that real life observations on economic variables are in most cases plagued by one or all of the problems of nonspherical disturbances and measurement errors.

Estimation Methods for Simulation Study

For simulation study, the following estimation methods were used: Ordinary least squares (OLS) estimator , Ordinary least squares corrected for autoregression (OLSAR) , Ordinary least squares corrected for moving average (OLSMA), Ordinary least squares corrected for (ARMA) autoregressive moving average (OLSARMA), Two-stage least square (2SLS), Two stage least square corrected of autoregression (2SLSAR), Two stage least square corrected for moving average (2SLSMA), Two stages least square corrected for autoregressive moving average (2SLSARMA), Asymptotically, each of these estimators are equivalent with identical asymptotic properties.

The Model and Design the Basic Model and the Independent Variable Specifications

The basic model employed in our experiment is the Koyck (1954) model which is autoregressive distributed Lag model where the error term follows a first order autoregressive structure. The model is given by

$Y_t = \beta_1 + \beta_2 X_t + \beta_3 Y_{t-1} + V_t$

$V_t = U_t - \lambda U_{t-1}$ and $V_t = \ell U_{t-1} + e_t$

This results to an autoregressive Xt as the independent variable for the experiment

$X_t = \ell X_{t-1}$ where $\ell = 0.8$

The selected independent variables are

i. Commonly used in applied work.
ii. Popular in the literature and have already been used in other Monte Carlo studies to facilitate comparison of the results with previous findings but the choice of model have not been used in the literature in order to make contributions in that area.

The Choice of Parameters and Specifications of the Independent Variable

The values of the parameters β1 , β2 , and β3 in the model

$$Y t = \beta_1 + \beta 2\ Xt + \beta 3\ Y_{t-1} + V t$$

$$V t = Ut - \lambda U_{t-1} : U t = \ell U_{t-1} + e_t \quad /\ell/ < 1$$

are arbitrarily chosen to be (1,1,1). The choice of the independent variable is equally important in Monte Carlo experiment. This is because an appropriate X matrix can allow meaningful, generalization of the results of the experiments.

Kadiyala (1970) and other have discussed in their works that the nature of the independent variables affects the efficiency of the estimators, therefore, to obtain the X variable with real life quality, the following procedures are followed. $Xt = 0.8X_{t-1}$ and the initial value X_0 is fixed arbitrarily as 515. This was the popular model used by Adiele and Nwabueze (2010).

The Generation of Sample Data

In order to generate multivariate normal vectors to be used, the autoregression error term U t = ℓUt-1 + e t and V t = U t – U t-1 was first generated as follows, though an E- view computer program was used.

1. Generate 1000 random values using E-view

2. For a replication of 20, repeat step 1 21 times to generate 21 different random samples each 1000.

3. Standardize each of the groups of the random values to get 21 different standard normal values of size 1000 each having mean O and variance 1. A starting value U_0 was generated by drawing a random e_1 from N (0,1) and dividing by $\sqrt{1 - P_2}$.

4. Successive values of e_t drawn in step 3 from N(0,1) were used to calculate U t = (U t-1 + E t and then V t = U t – U t-1 and the first sample was thrown away to avoid problems of initial values. 20 samples at size 1000 each were generated for each ρ in step 4.

For sample size of 40 and 60 the steps were repeated. Estimates of rho were used at $\rho = 0.4$, 0.6 and 0.8.

After generating the autoregressive error term for each specification, multivariate normal Y vectors are generated using the equation.

$Y_t = \beta_1 + \beta_2 X_t + \beta_3 Y_{t-1} + V_t$

The computations are made using a software package, which is available in the internet.

Different estimation methods are applied to the data using E. view.

Analysis of the Results

This study employed the minimum Bias criteria to evaluate and compare the estimators. We also use the sum of Biases (SBIAS) to evaluate the methods. Consider an estimator $\beta = (\beta_0, \beta_1, \beta_2)$. The SBIAS of β is attained as the sum of Biases of $= \beta_0$, β_1 and β_2 .

Simulation Results for Estimators

The summary of principal calculations for each model, estimation procedure, degree of autocorrelation of the error term and each sample size are presented in tables on the appendix, these include the bias (BIAS), variance (VAR). Also calculated and displayed are the absolute sums of bias (SBIAS).
For any parameter, the ith estimate is denoted by β_i and the true value by β_i .
Therefore:

$$\text{BIAS}(\hat{\beta}_i) = \frac{1}{1000} \sum_{i=1}^{1000} (\hat{\beta}i - \hat{\beta}i) \qquad (4.1)$$

where

$$\hat{\beta}_i = \frac{1}{1000} \sum_{i=1}^{1000} \hat{\beta}i$$

BIAS for Estimators of β in Model when

$Y = \beta_0 + \beta_1 X_t + \beta_2 Y_{t-1} + U_t$ $\quad$ $R = 1000$, $\beta_0, \beta_1, \beta_2 = (1,1,1)$
$X_t = \lambda X_{t-1} + e_t$

n = 20, ℓ = 0.4, Average Bias

Estimators	β 0	β 1	β 2	SBIAS
OLS	-0.253860301987	-0.166748342865	0.0129032172153	-0.407705125
OLS-AR	-0.382653	-0.089086	0.034342	-0.43739609
OLS-MA	-0.5213881	-0.0880390	0.04985744	-0.55956966
OLS-ARMA	-0.1406302	-0.0948305	0.02080134	-0.21465936
2SLS	-2.1510694	0.5955585	0.27348957	-1.49668049
2SLS-AR	-1.8041316	0.4555979	0.2254337	-1.1231000
2SLS-MA	-2.0467127	0.499178217	0.261836522	-1.285697961
2SLS-ARMA	-2.00178628	0.577648826	0.2596564135	-1.164481041

n = 20, ℓ = 0.6, Average Bias

Estimators	β 0	β 1	β 2	SBIAS
OLS	0.0128742463424	-0.276797476892	-0.0274362550795	0.291359485
OLS-AR	-0.492558106109	-0.0631466138879	0.0451418040974	0.510562915
OLS-MA	-0.527967050628	-0.0907706843381	0.0462759432267	0.572461791
OLS-ARMA	-0.3562088992	-0.103811945262	0.0271427111541	0.432878126
2SLS	-2.90977608116	0.846353792557	0.33000233519	1.733419953
2SLS-AR	-2.71347030341	0.736809988758	0.306887047393	1.669773267
2SLS-MA	-2.86460920901	0.759828342756	0.324631334791	1.780149532
2SLS-ARMA	1-2.87020921046	0. 837270387672	0. 328100612459	1.70483821

n = 20, ℓ = 0.8, Average Bias

Estimators	β 0	β 1	β 2	SBIAS
OLS	2.50486905171	-1.06441413878	-0.337022241376	1.103432672
OLS-AR	0.226617704516	-0.133082233081	-0.0363024786565	0.057232992
OLS-MA	0.464882941196	-0.3886413327773	-0.0773756847264	1.13407632
OLS-ARMA	2.06890286693	-0.471150674896	-0.235904781974	1.361847409
2SLS	-50.6269043794	13.2123145782	6.30806635726	-31.10652344
2SLS-AR	-50.8357654073	13.2922542176	6.32794993513	-31.21556126
2SLS-MA	51.0023056395	13.369085468	6.35451395416	70.72590504
2SLS-ARMA	-51.0584300256	13.5421303466	6.36974001465	-31.14655967

n = 40, ℓ = 0.4, Average Bias

Estimators	β 0	β 1	β 2	SBIAS
OLS	-0.407634159047	0.052815727411	0.0271222502745	0.327696181
OLS-AR	-0.454311479649	0.0523134838656	0.0303877543736	0.371610241
OLS-MA	-0.481225268458	0.0492238851693	0.0321106629193	0.39989072
OLS-ARMA	-0.453662501286	0.0604248628754	0.030606063446	0.362631574
2SLS	25.4483459836	1.89341457832	-1.29508090786	26.04667965
2SLS-AR	15.4081422684	-0.404482767336	-1.70640197876	13.29725751
2SLS-MA	28.7893856777	2.8001504483	-1.50155836757	30.08797775
2SLS-ARMA	-10.0480605358	-0.861748653387	-1.38500487741	9.524804287

n = 40, ℓ = 0.6, Average Bias

Estimators	β 0	β 1	β 2	SBIAS
OLS	-0.440382762371	0.0748028953178	0.337826143	0.0277537233386
OLS-AR	-0.523389848808	0.0566988060499	0.433621671	0.0330693709089
OLS-MA	-0.545222267282	0.054618170879	0.45614565	0.0344584460282
OLS-ARMA	-0.529266636188	0.0586004604456	0.437160042	0.0335061328818
2SLS	26.2984479363	2.12343249643	26.59999397	-1.82188645915
2SLS-AR	31.4312333564	0.207598361375	29.47829684	-2.16053487341
2SLS-MA	29.1845541377	2.96964153215	30.15495192	-1.99924374679
2SLS-ARMA	-3.32478740924	1.28679802676	3.951511024	-1.81352164157

n = 40, ℓ = 0.8, Average Bias

Estimators	β 0	β 1	β 2	SBIAS
OLS	-0.295133139738	0.13403213646	0.0165322819013	0.144568721
OLS-AR	-0.545452803392	0.0513253307894	0.0313959174594	0.462731555
OLS-MA	-0.564377238172	0.0525548007294	0.0325875418586	0.479234895
OLS-ARMA	-0.553862299677	0.0491293840935	0.0319045043174	0.472828411
2SLS	6.85371467295	0.332407922164	-0.389840280467	6.796282314
2SLS-AR	5.47400443364	-1.25468624866	-0.543940409245	3.675377261
2SLS-MA	8.55253348988	0.981226490989	-0.482824048034	9.050935932
2SLS-RMA	1.3090168547	-0.934307389924	-0.451322459073	0.076612994

As a way of illustration, to get the relative efficiency in Table 2 from Table 4.2, we have Var β(OLS) as 0.4004 and Var β(OLS-AR) as 0.6480, so; the efficiency of OLS-AR for

$$\hat{\beta}_0 = \frac{0.4004}{0.6480} = 0.62 \quad \text{i.e.}$$

$$\frac{\text{Var } \hat{\beta}_0 \text{ (OLS)}}{\text{Var } \hat{\beta}_0 \text{ (OLS - AR)}} = 0.62$$

Also for $\hat{\beta}_1$ have $\frac{0.120150527476}{0.102244980914} = 1.18$

$\hat{\beta}_1$ have $\frac{0.00782593041163}{0.0701498963917} = 1.11$

Then to get the total gain or loss we subtract one from each efficiency and add our results i.e. for OLS – AR in the Koyck model under variance for n = 20, P = 0.4. it is (0.62 - 1) + (1.18 - 1) + (0.11 - 1) = -1.09, showing that we have a loss of about 109%. If we use OLS - AR instead of OLS to estimate the koyck model based on the variance criterion.

Table 2: Efficiency of the Estimators Relative to OLS Based on VAR in the Koyck Model

Y = β0 + β1 X t + β2 Y t-1 + V t, R = 1000, β0, β1, β2 = (1, 1, 1)

n = 20, ℓ = 0.4,

ℓ	Estimators	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	G/L
	OLS	1.00	1.00	1.00	0.00
	OLS-AR	0.62	1.18	0.11	-1.09
	OLS-MA	0.06	0.97	0.90	-0.43
0.4	OLS-ARMA	0.06	0.98	0.43	-0.53
	2SLS	0.0018	0.0043	0.0014	-3.00
	2SLS-AR	0.0025	0.0064	0.0019	-3.00
	2SLS-MA	0.0019	0.0044	0.0014	-3.00
	2SLS-ARMA	0.0019	0.0044	0.0014	-3.00

		ℓ = 0.6,			
	OLS	1.00	1.00	1.00	0.00
	OLS-AR	1.74	1.78	2.16	2.68
	OLS-MA	1.36	1.45	1.71	1.52
0.6	OLS-ARMA	0.67	1.19	0.91	-0.23
	2SLS	0.0008	0.0024	0.0017	-3.00
	2SLS-AR	0.0092	0.0029	0.0020	-3.00
	2SLS-MA	0.00083	0.0024	0.0017	-3.00
	2SLS-ARMA	0.00082	0.0024	0.0016	-3.00
		ℓ = 0.8			
	OLS	1.00	1.00	1.00	0.00
	OLS-AR	0.49	5.06	0.71	3.26
	OLS-MA	2.85	2.84	2.83	5.53
0.8	OLS-ARMA	0.20	1.14	0.28	-1.38
	2SLS	0.000021	0.0000028	0.0000020	-3.00
	2SLS-AR	2.1 -05	2.7 -05	2.0 -05	-3.00
	2SLS-MA	2.1 -05	2.8 -05	2.0 -05	-3.00
	2SLS-ARMA	-05 2.1	2.8 -05	2.0 -05	-3.00
		n = 40, ℓ = 0.4,			
ℓ	**Estimators**	$\widehat{\boldsymbol{\beta}}_0$	$\widehat{\boldsymbol{\beta}}_1$	$\widehat{\boldsymbol{\beta}}_2$	**G/L**
	OLS	1.00	1.00	1.00	0.00
	OLS-AR	0.98	1.06	0.90	-0.06
	OLS-MA	0.94	0.97	0.85	-0.24
0.4	OLS-ARMA	0.69	0.81	0.64	-0.86
	2SLS	2.2 -07	4.9 -06	1.6 -07	-3.00
	2SLS-AR	2.0 -07	4.0 -06	2.0 -07	-3.00
	2SLS-MA	2.0 -07	3.0 -06	4.0 -07	-3.00
	2SLS-ARMA	-07 2.0	3.0 -06	4.0 -07	-3.00
		ℓ = 0.6			
	OLS	1.00	1.00	1.00	0.00
	OLS-AR	0.72	1.34	0.72	-0.22
	OLS-MA	0.57	1.13	0.54	-0.76
	OLS-ARMA	0.51	1.11	0.45	-0.93
0.6	2SLS	2.59 -08	1.70 -06	1.59 -08	-3.00
	2SLS-AR	2.50 -08	1.69 -06	1.59 -08	-3.00
	2SLS-MA	2.59 -08	1.70 -06	1.59 -08	-3.00
	2SLS-ARMA	1.98 -08	1.69 -06	1.59 -08	-3.00

			ℓ = 0.8		
0.8	OLS	1.00	1.00	1.00	0.00
	OLS-AR	5.55	2.13	0.70	0.38
	OLS-MA	3.30	1.54	3.67	2.51
	OLS-ARMA	3.37	1.82	5.25	4.42
	2SLS	2.04 -07	3.84 -05	2.97 -06	-3.00
	2SLS-AR	1.24 -03	3.64 -05	2.97 -06	-3.00
	2SLS-MA	2.04 -07	3.81 -05	2.97 -06	-3.00
	2SLS-ARMA	1.62 -07	3.66 -05	8.43 -09	-3.00

Findings and Conclusion

Monte Carlo experiment yield results which are of practical and methodological relevance in conducting empirical studies. This work has shown that 2SLS, 2SLS-AR, 2SLS-MA, and 2SLS-ARMA estimators possess similar behavioral patterns and characteristics, similarly, it was found that the OLS-AR and the OLS-ARMA estimators are quite comparable in performance in estimating Koyck model with autocorrelated error terms when the error term is small and when the error term is high. The Monte Carlo evidence shows that while the effect of increasing sample size on the comparative performance of the estimators is of some slight significance, the effect of increasing autocorrelation of the error terms on the comparative performance of the estimators are remarkably significant. However, the result also revealed that as the sample size increases from 20, to 40, some of the conclusions arrived at in this study become increasingly significant.

It is difficult to make a conclusive statement regarding the performance of the estimators because their comparative performance depends on a number of factors including the size of autocorrelation, the property of the estimator and the individual coefficient being estimated. However, a surprising result is that of finding that some of the OLS estimators which correct for autocorrelation perform worse than the OLS which does not correct for autocorrelation at all. On a prior ground, one would expect these estimators to be superior to OLS after it has transformed the data.

However, the finding is not unique in this regard since Nwabueze (1982) observed similar results in their experiments. The argument in this direction is that the transformed variable for some models is such that transformation with a positive value of ℓ tends to decrease the variability of the independent variable. More specifically OLS performs better than OLS-AR, OLS-MA, 2SLS, 2SLS-AR, 2SLS-MA, 2SLS-ARMA and for small sample and low ℓ OLS also performs better than OLS-ARMA, for increased sample size and ℓ OLS is exactly the same as OLS-ARMA. However, OLS-AR, OLS-MA, OLS-ARMA, perform better than OLS in estimating the β1 coefficient. In this work the variance and Root Mean Square Error for β1 in this model tend to increase very slowly with increasing autocorrelation of the error term while those of β1 (OLS) increase sharply with increasing autocorrelation.

It is further observed from the results for large samples n = 40 that the finite sample properties of the estimators approximate their asymptotic properties fairly well but much also depends on the size of autocorrelation of the error terms, and the individual coefficient being estimated.

The experiments revealed that the most preferred β0 (.) does not lead to the most preferred β1 (.) or β2 (.) under all the finite properties considered. It was noticed that increases in the size of autocorrelation affect the performance of the estimators differently in the estimation of β0 (.), β1 (.) and β2 (.). The comparative performance of the estimators in estimating β0 (.), β1 (.) and β2 (.) differs from ℓ specification to ℓ specification. These findings concerning β0 (.), β1 (.) and ˆ β2 (.) are ˆ known and their finite sampling properties are not known, these Monte Carlo results can guide the choice of estimators in empirical work. Therefore, it is not plausible in practice to base the decision to choose a particular β0 (.) because of the known sampling properties of β1 (.) or β2 (.) even with the same estimator. It rather suggested that a decision to choose a particular estimator to estimate β(.) should be based on established sampling properties of β1 (.) and β(.). However, it was discovered that OLS dominated when n is small and P is small, OLS n increases and ρ increases OLS and OLS-AR complete favourably.

In this study observed a number of unexpected or surprising results. Some of the findings that do not follow conclusive pattern clearly reveal that the ˆ search for best estimators of models plagued by autocorrelated disturbances could be hazardous. These do not only vary with the degree of autocorrelation of the error terms but also they vary with the criteria chosen, the size of the sample and the individual coefficient being estimated. These arguments are suggestive of the fact that studies which make use of more replications under range of sample sizes and parameter values are needed on these issues so that conclusions could be guided by consistent empirical evidences.

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