

MAXIMUM BIAxIAL AND SUPERIMPOSED SHEAR BEHAVIOUR OF ARCHITECTURAL GRADE SINGLE-PLY FABRICS

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Abstract

An expression for shear stress/strain, which considers the rotation of oiie axis of fabric yarn, i.e. the warp axis, is derived. This expression is considered sufficiently accurate within the range of strains as encountered in this study. Stress/strain characteristics and shear hysteresis diagrams are presented for fabric cylinders subject to multiaxial stresses of varying ratios. These diagrams are shown to be exclusively dependent on the ratio of stresses in the orthogonal directions. The behaviour is also shown not to obey Hookean elasticity as most mathematical models assume.

Introduction

The shear characteristics of woven fabrics play an important role in the overall mechanical behaviour. Haas (1917) pioneered this aspect by a study of shear in a plain weave fabric subjected to multidirectional stress fields. This was achieved by twisting a pressurized cylinder. The inflated cylinder was subjected to torque at one end with the other piece restrained against rotation. The deformation was then measured by drawing a rhombus standing on one end (with its diagonal parallel to the axis of the cylinder).

Apart from twisting a pressurized cylinder, a qualitative assessment of the shear behaviour of fabrics may be achieved by tests on a strip. The strip may be cut in the bias direction so that the applied tension is resolved into warp, weft and shear stresses. But as mentioned in the author's earlier paper Nwankwo (2002), the stress distribution is complicated by the grip and the fact that the ends of yarns, i.e. the edges, are not restrained and therefore cannot sustain any form of loading. This last difficulty may be solved by applying a pretension to the square fabric before shearing. M©rner and Eeg-Olofsson (1957) and Behre (1961) confirmed the non-uniform stress distribution associated with this technique by testing a square fabric specimen. Cusick (1961) also studied fabric drape using fabric shearing apparatus developed by Morner and Eeg-Olofsson. These investigators attempted to establish the relationship between the shearing forces applied to two clamps on the opposite side of the fabric specimen and the deformation produced. Investigators such as Behre suggest a parabolic stress distribution but the author here disputes this. Parabolic stress distribution can only be justified for a homogenous isotropic thin plate.

Tests on pressurized cylinders with the test rig as detailed by Nwankwo (1982), and Nwankwo (2002) are probably the best methods of simulating the shear behaviour of fabrics since there are no free edges. A disadvantage however is the non-uniform strain distribution due to seam. In the author's opinion and as shown in the author's previous paper (Nwankwo 2002), this may be neglected if the seam stiffness is made such that it is nearly the same stiffness as the parent fabric specimen. Another disadvantage is the end effects both at lower and upper ends of the inflated cylinder specimen, where it is clamped. This can be remedied by selecting a specimen of sufficient length to diameter ratio. Yendell (1971) carried out a thorough mathematical analysis to determine the distance from both ends of the inflated cylinder at which the end constraints have negligible influence on the measured strain, i.e. region of controlled strain. This analysis requires numerical solution of first and second order differential equations.

Analysis of Shear Stress

The fabric cylinder test apparatus used in this study, is the same as that fully described in the author's earlier paper, (Nwankwo 2002). The yarns of the fabric specimen are composed of glass fibre which are woven into cloth and coated with polytetrafluorethylene (P.T.F.E.) and other fillers. The preparation and size of specimens are as described in the above paper. This apparatus is designed to induce a shear stress in the fabric by twisting the upper fabric end clamp (upper disc) by means of loads applied to cables attached to the rim of the clamp, with the lower end of the fabric of cylinder

Let the applied tension be	T
Shear stress	t
Thickness of disc Thickness of disc	t
Diameter of disc	D
Diameter of fabric cylinder	d

Referring to figure 1:

restrained against rotation. This upper disc is attached to a steel shaft incorporating a set of ball bearings so that the disc can rotate independently.

To derive the shear stress consider the free body diagram of this upper disc shown in figure 1

$$M(T \cdot \frac{D}{2}, C) = t(D \cdot t)$$

Similarly

$$M = 0.2 \cdot r \cdot r \quad \text{or} \quad M = t \cdot C \cdot r$$

Where C = circumference of fabric cylinder So that $T(D/2) = t \cdot C \cdot r$ 1

In equation 1, either the applied tension or the shear stress can be treated as the unknown. Thus, either can be deduced from a knowledge of the other.

In some of the shear tests in the present study, it was necessary to use equation 1 to calculate the applied tension equivalent to a specified shear stress. This is the case where it is required to observe the behaviour of the fabric cylinder as the critical shear stress is approached. In other cases, in order to avoid premature failure, it was necessary to restrict the maximum shear angle. In these cases the tension for a particular load level can be specified and the induced shear stresses calculated.

Analysis of Shear Strain

In attempting to derive an expression for shear strains in materials subjected to two-dimensional biaxial strain, consideration must be given to the applicability of such expressions to experimental studies. Certain assumptions are therefore necessary. It will be assumed here that for a fabric sheet subjected to a system of multidirectional strains the component of shear strain can be expressed as a linear function of the strains in the warp and weft directions. Thus, the angle through which the warp and weft yarns rotate relative to each other will be the shear strain. To formulate an expression describing the state of stress where large strains are involved, requires that the rotation of the strain axis be considered (i.e. the strain is measured in the direction of stress). However, this may be ignored where strains are small, such as in metals since the rotation of the strain axis will be negligible. An analysis which considers the rotation of both strain axes (i.e. rotation of the warp and weft axis) will not be easily applicable. The analysis presented here therefore assumes the weft axis is fixed in direction and the shear strain is derived as follows:

Figures 2 (a) and (b) represent a triangular shaped sheet of fabric of unit dimensions in the unstrained and strained state respectively. Application of the cosine rule to the strained triangle gives: $N^2(1 + c_{bias})^2 = (1 + e_{warp})^2 + (1 + e_{wcn})^2 - 2(1 + e_{warp})(1 + e_{wcn}) \cos(\theta/2 + \alpha)$

$$2(1 + c_{bias})^2 = (1 + e_{warp})^2 + (1 + e_{wcn})^2 + 2(1 + e_{warp})(1 + e_{wcn}) \sin \alpha$$

$$\text{So that } \alpha = \frac{2(1 + c_{bias})^2 - (1 + e_{warp})^2 - (1 + e_{wcn})^2}{2(1 + e_{warp})(1 + e_{wcn})}$$

Equation 2 may be used in its present form, or it may be further simplified by expanding $\sin \alpha$ by power series and neglecting products of small quantities such as $e_{warp}^2, c_{wcn}^2, \sin \alpha, c_{warp}^2, \alpha^3$ etc. The magnitudes of strains likely to be encountered in this study are $e_{warp} = 1.5\%$, $c_{wcn} = 3\%$ and $c_{bias} = 15\%$. For these values of strains, the maximum error introduced by neglecting the higher order terms in the series is approximately 8% shear strain, so that equation 2 becomes: $\alpha \approx \frac{2c_{bias}}{1 + e_{warp} + e_{wcn}}$

There would be no appreciable difference when either of the equations are used in calculating shear strain.

An alternative way of measuring the shear strain is by observing the rotation of the topclamping disc. The outer circumference of the upper disc is graduated in centimeters and millimeters. A pointer attached to the support framework indicates the rotational displacements from which the shear strains can be calculated. The displacements at this stage are relative to the disc and must

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therefore be corrected to take account of the distance from the fabric to the outer circumference of the disc. The correction factor to be applied is derived as follows:

Figure 3 shows the outline of the disc with (he fabric specimen clamped).

From figure 3:

$$X = \frac{Y_j}{(r + .R)}$$

Where y = Recorded rotational displacement with initial reading subtracted.
R = radius of fabric specimen
.R = width from inner face of disc to the outer face
x = corrected displacement.

Shear angles were measured by this method and also by attaching Vernier scales to the fabric specimen. The correlation of the two forms of measurements, not presented here is found satisfactory.

Test Apparatus and Mode of Loading

The test apparatus adopted in this study is co-developed and detailed in the author's previous work [Nwankwo 1982] and housed in the Wilfred Merchant Heavy Structures Laboratory at the Faculty of Technology of the University of Manchester in England.

A total of ten fabric cylindrical specimens were stressed in increments to the maximum principal stress, then torque is applied in increments.. Two pairs of rosettes of gauges were attached to the fabric specimen to measure strains. Figure 4 shows the location of these gauges on an unfolded test piece. The titles of the figures presenting the results graphically are given in Table 1.

Analysis and Discussion of Results

The general modes of deformation involved can be seen from the shear resistance diagrams. A point to observe is the relationship between the shape of the shear resistance diagrams, for different ratios of principal stresses in the warp and weft directions. Specimen 'C' was subjected to clockwise torque only, while specimens 'B' and 'H' were subjected to both clockwise and anti-clockwise torque. There is no curve for the fabric response to anti-clockwise torque for specimen 11 because the specimens failed prematurely at the boundary of the seam and parent fabric the same position as the failure in specimen B. The failure load is unlikely to be the full shear capacity of the fabric. Plate 1 gives a full view of the twisted cylinder showing the tear for specimen B. These failures are attributed to stress concentration in the parent fabric where it meets with the seam fabric owing to the difference in stiffness and the effects of fabricating the end of flaps and seam. The tears propagated from small tears at the positions marked 'y' and 'z' in plate 1.

Conclusion

Measurement of the shear angle by the rotation made by top disc of the test rig gave values in close agreement with those measured by the Vernier scales which were bonded to the fabric specimens. It is therefore not essential to attach Vernier scales in the bias direction of the fabric specimen, thus reducing the number of attachments.

The mechanism of shear deformation is seen to be a function of the applied shear stress and three basic modes of deformation are identified viz.: thread shear, thread straightening and thread extension.

The shear resistance diagram is dependent on the frictional restraint at yarn intersection due to rotation of the warp and weft hence the biaxial stress ratio of the warp and weft and also the magnitude of applied torque.

References

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Table 1: Title of Figures, Specimen Identity and Stress Ratios:			
Figure	Title	Specimen Identity	Stress Ratio (Warp/Weft)
5	Warp and Weft response to loading	B	1:1
6	Shear response (shear angles measured by top disc)	B	1:1
7	Warp response to loading and unloading	C	1:2
8	Weft response to loading and unloading	C	1:2
9	Shear response (shear angles measured by top disc)	C	1:2
10	Shear response (opening and closing shear angles measured by rosettes marked 'B', 'D' and 'A', 'C' respectively in Figure.4 (b).	C	1:2
11	Warp response to loading	H	1:3.75
12	Weft response to loading	11	1:3.75
13	Shear response (shear angle measured by top disc)	11	1:3.75
14	Shear response (opening and closing shear angles measured by rosettes marked 'A', 'D' and 'B', 'C' respectively in Figure 4 (c).	H	1:3.75
15	Relationship between warp and weft strain (Specimens B, C and H)	11	1:3.75

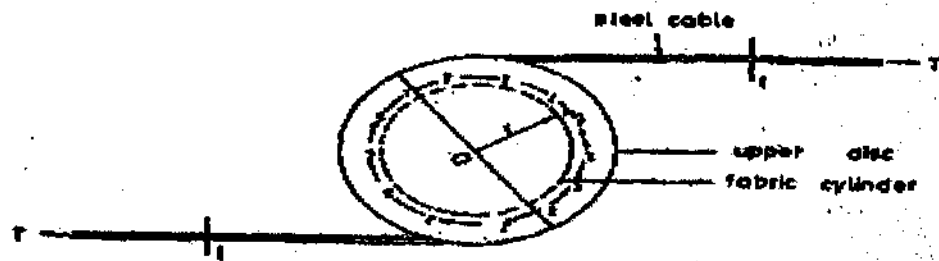
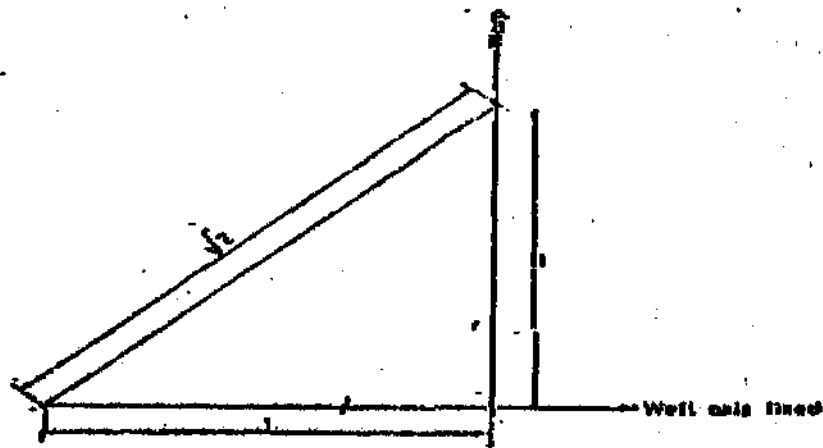


Figure 1. Forces acting on fabric cylinder subjected to torque.



(a) unstrained

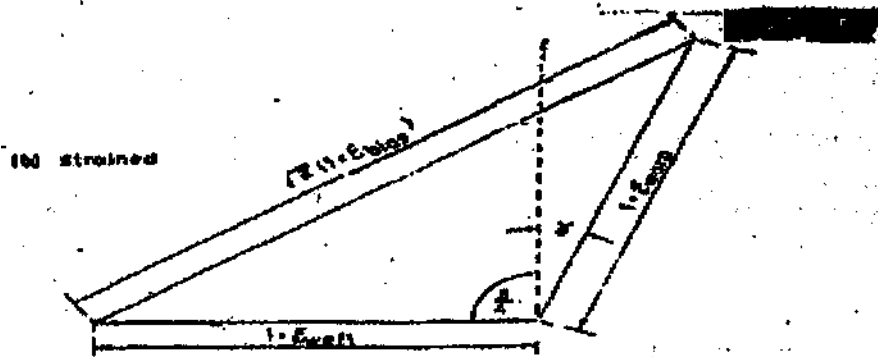


Figure 2. Element of fabric sheet.

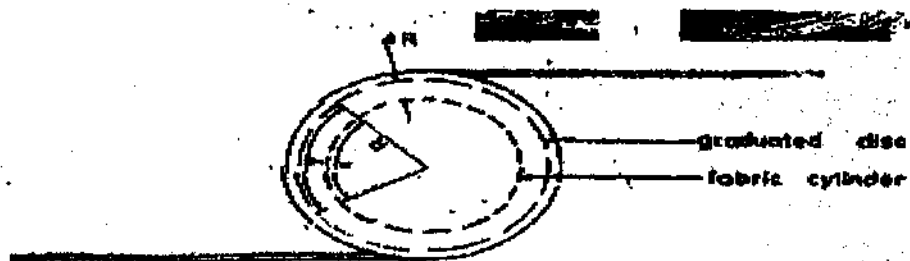


Figure 3. Outline of disc with fabric specimen stamped.

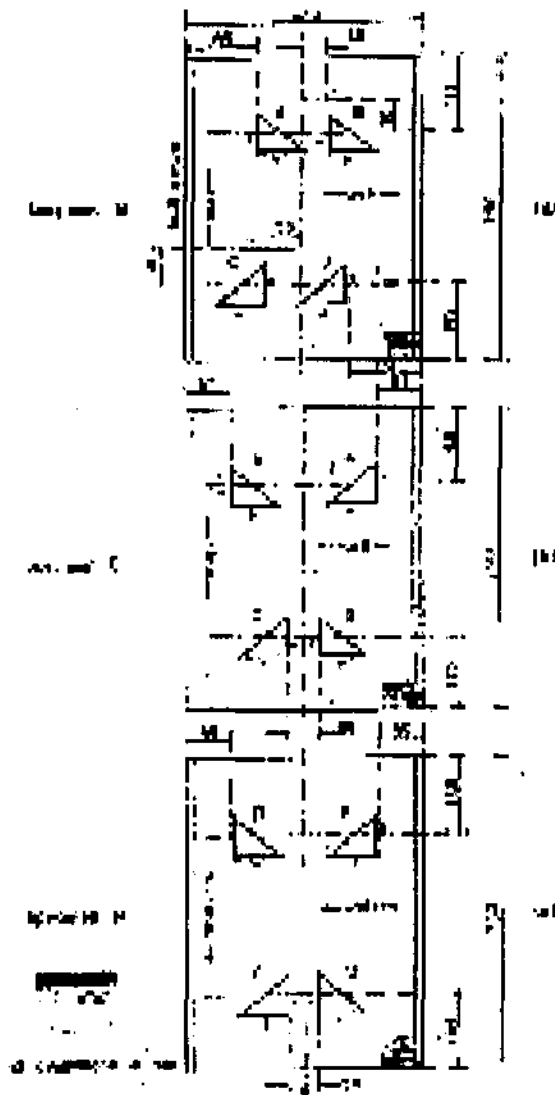


Fig. 1: Schematic diagram.

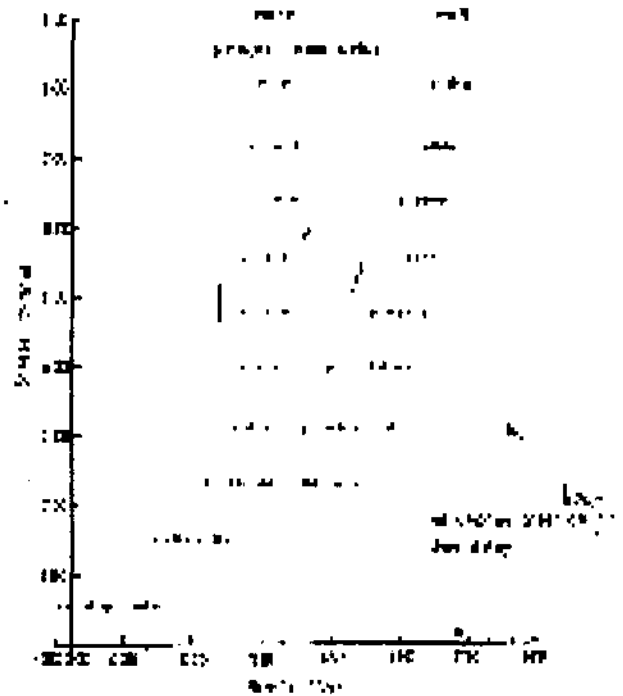


Fig. 2: Relative Humidity vs. Relative Humidity.

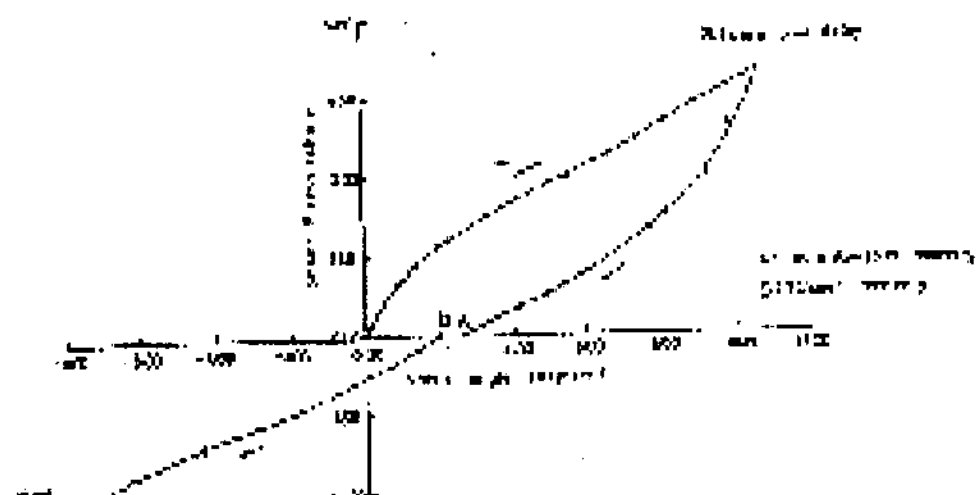


Fig. 3: Relative Humidity vs. Relative Humidity.

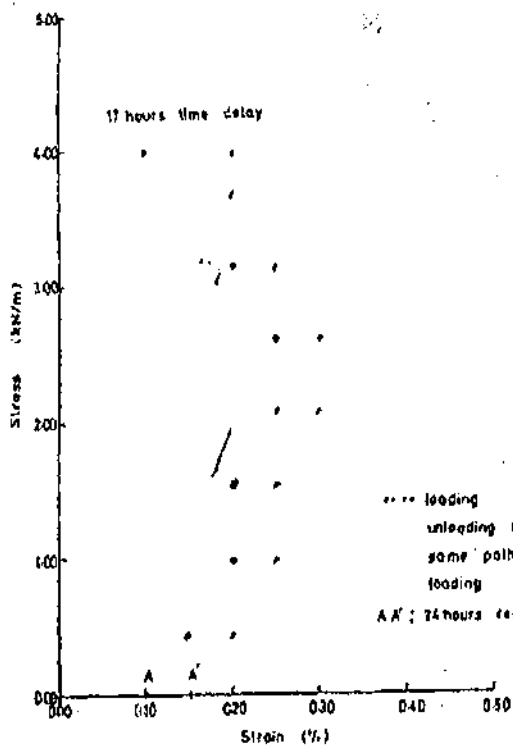


Figure 7. Warp response to loading and unloading
Specimen C S.R. = 1:1

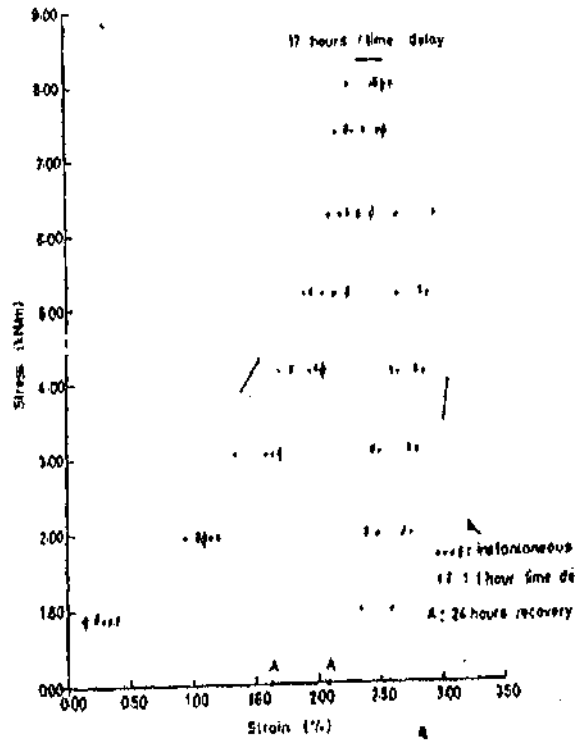


Figure 8. Weft response to loading and unloading
Specimen C S.R. = 1:2

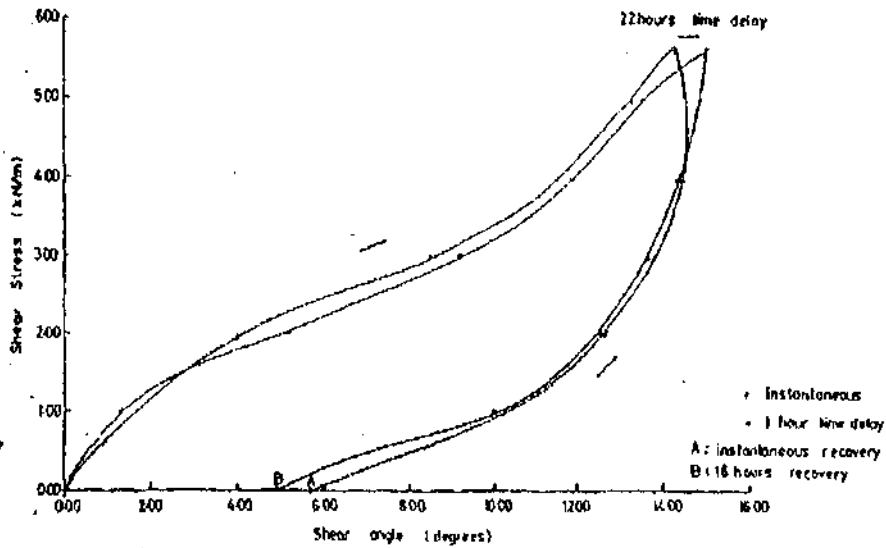


Figure 9. Shear Resistance (Shear angle measured by top disc)
Specimen C Warp Stress = 4.000kN/m, Weft Stress = 8.000kN/m.

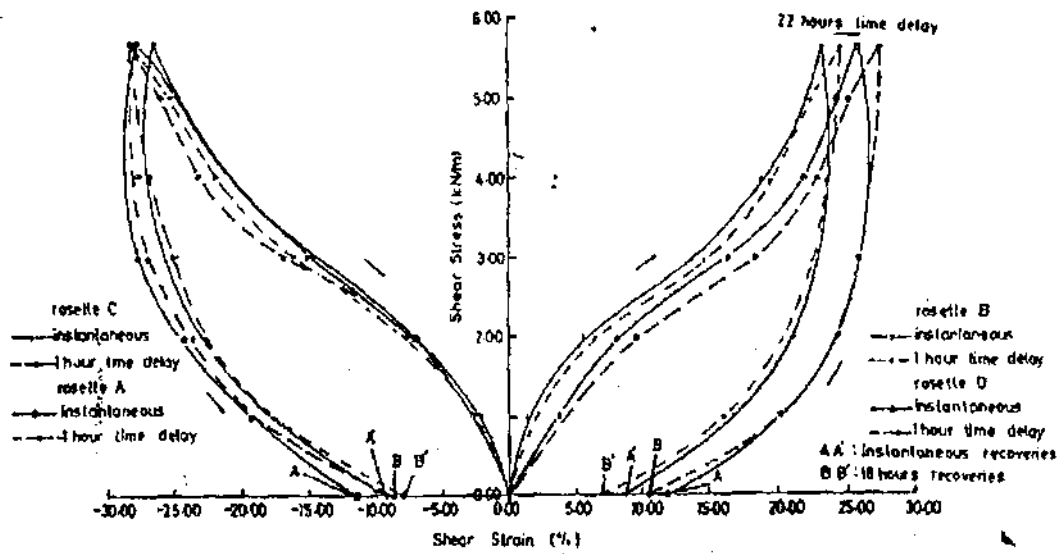


Figure 10. Shear Resistance
Specimen C Warp Stress = 4.000kN/m. Weft Stress = 8.000kN/m.

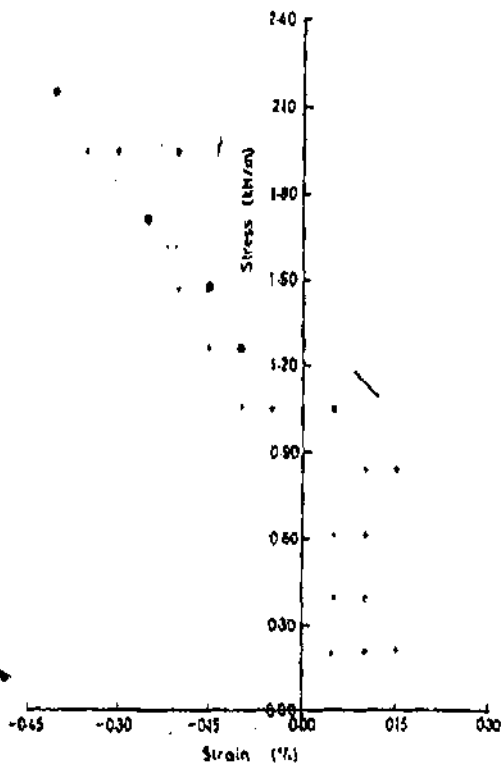


Figure 11. Warp response to loading
Specimen II S.R.=1:3.75

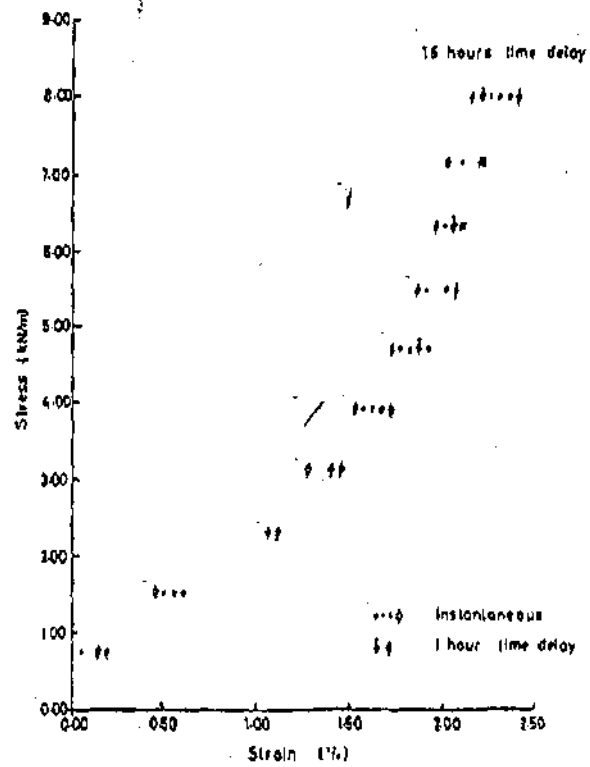


Figure 12. Weft response to loading
Specimen II S.R.=1:3.75

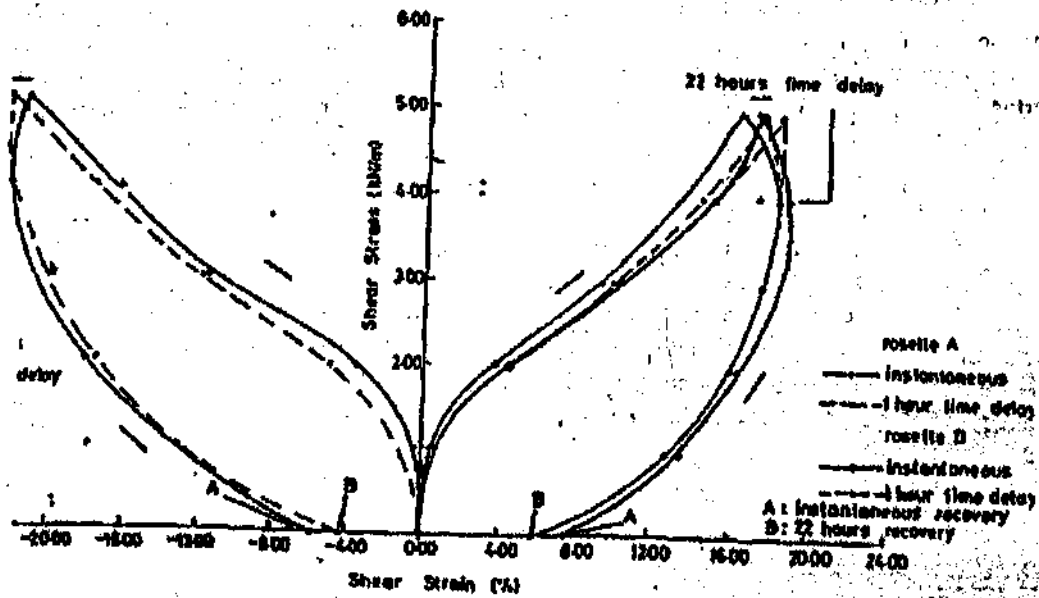
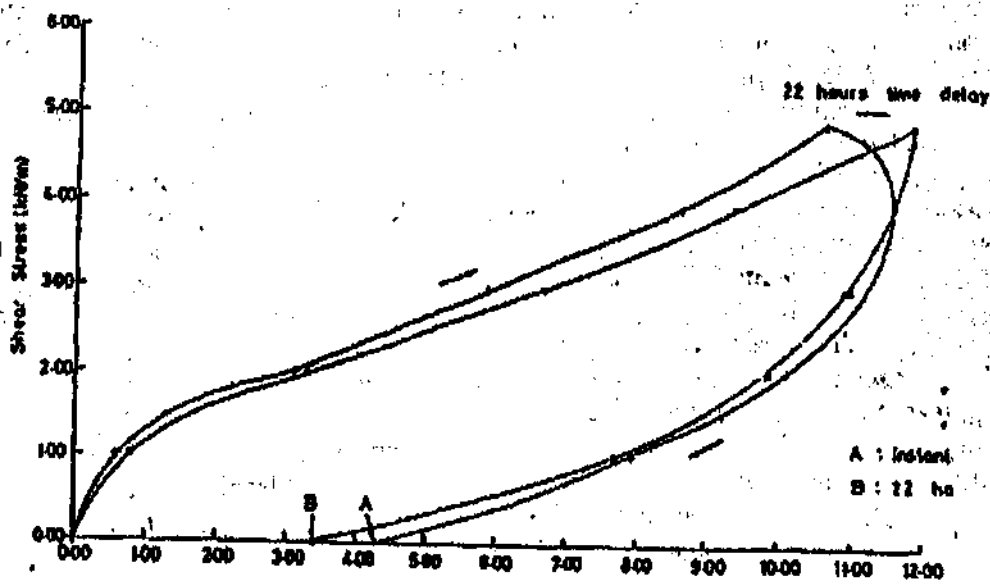


Figure 14. Shear Resistance
Specimen II Warp Stress = 2.667kN/m. Weft Stress = 8.000kN/m.

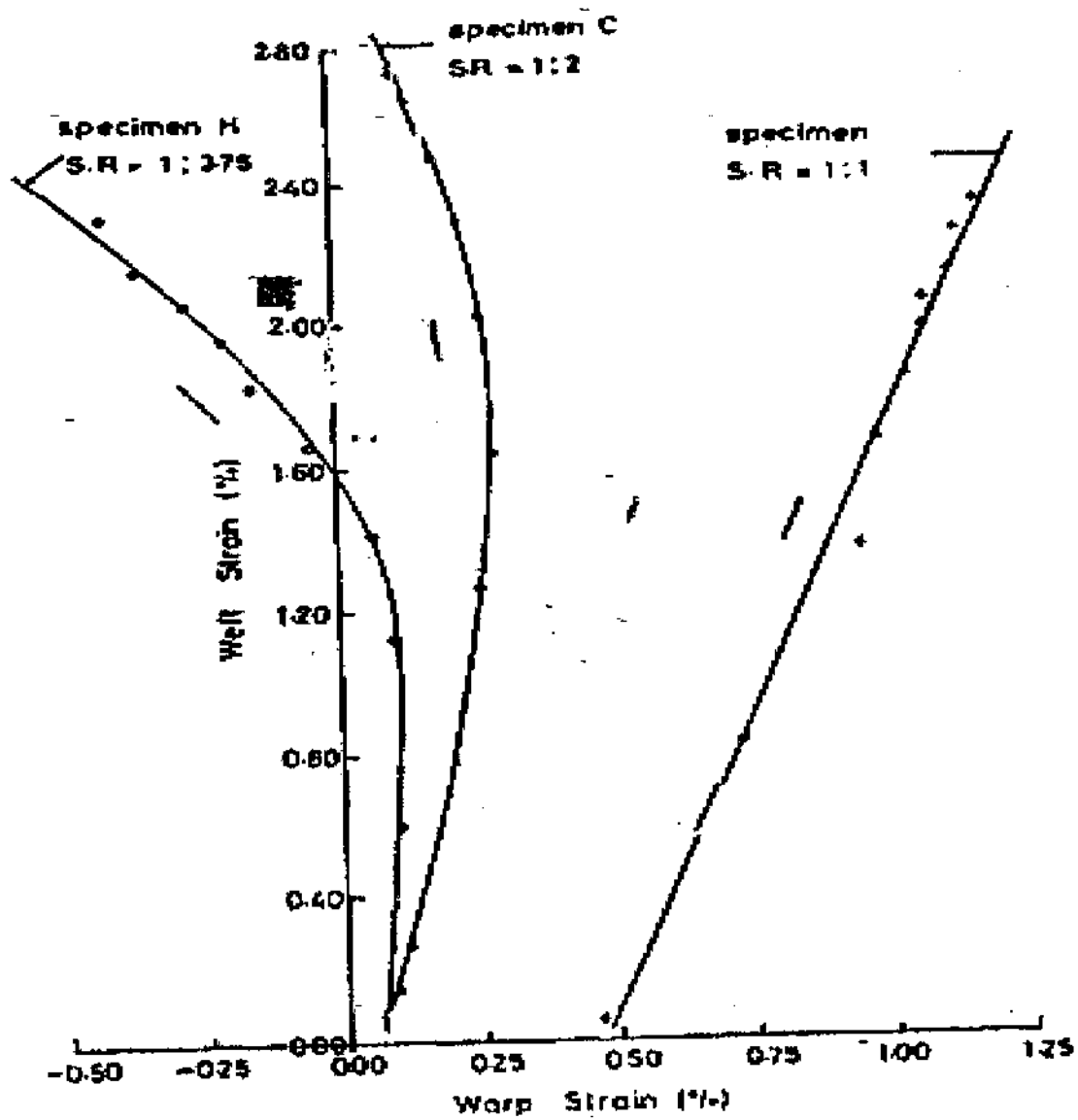


Figure 15. Relationship between Warp and Weft Strains
Specimens H, C and S



TOP



BOTTOM

Enlarged View

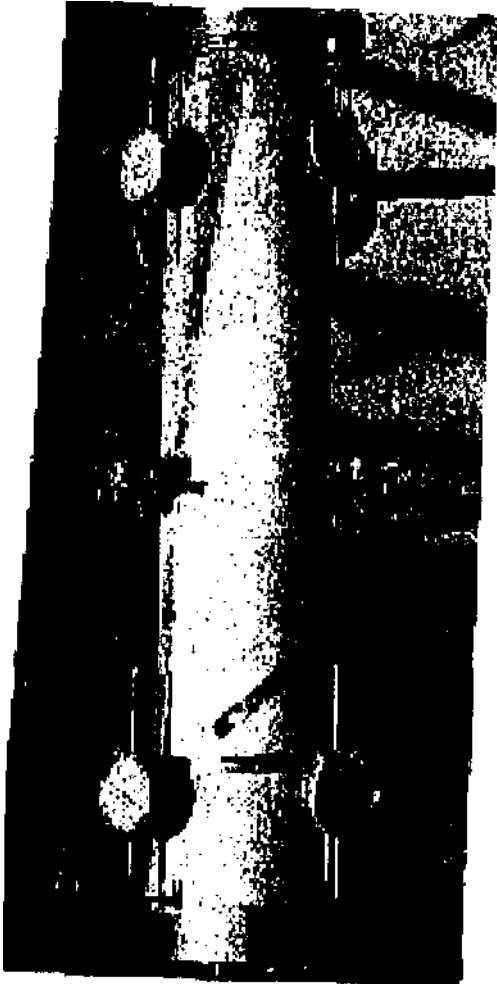


PLATE 1 FAILURE IN SHEAR OF SPECIMEN