APPLICATION OF MIXED-INTEGER LINEAR PROGRAMME (MILP) TO OPTIMIZE THE COST OF ENERGY IN THE SUPPLY OF WATER

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Abstract

Planning for an adequate water supply continues to be a challenge as water demands and supply reliability continue to change. The development of operational models of water supply systems that enlighten decision makers also continue to be a challenge, therefore, mathematical problems in which some of the variables are restricted to integer values called Mixed-Integer- Linear Programme (MILP) a logarithms for the energy cost optimization of Asejire Dam was developed. The objectives of this application is to simulate the cost that would have been incurred had the system been operated in accordance with the doctrine of MILP model. In applying the model, it was revealed that, the total energy cost reduces progressively and considerably as the storage level of the dam increases and a maximum of 25% decrease in the operating energy costs was also obtained.

Introduction

Water is essential to life and permeates most deeply into all aspects of our life out of all the natural resources. It is essential as the air we breathe for our very survival. Its presence or lack of determines to a great extent the nature of the natural environment in which we live and majority of our economic activities depends on it.

The development of water resources requires the conception, planning, design, construction and operation of facilities to control and utilize water. Water resources management is the general aspect of water supply to households and communities, irrigation, hydroelectric power development and other utilization of water for beneficial purposes.

Since planning is one of the most important task of management. It follows that planning for water resources management is a critical activity. The decision making process for the development of water resources management plan involves the optimization of water related resources by either maximizing or minimizing these resources under specific types of constraints that affect these resources and the outcome of the provision of water supply.

Throughout the history of the world, dams and reservoirs have been used successfully in collecting, storing and managing water needed to sustain civilization. Dams and reservoirs are integral part of our infrastructure and they can be compatible with the social and natural environment of watershed and region. The challenge for the future will be the utilization of dams and reservoirs in conjunction with the climate, environment and land use for the prudent management of the world's water resources as part of each nation's social and economic development goals.

Optimization, in general, deals with the allocation of limited resources with the objectives in mind of extremizing some outcome, its outcome is called objective function. It is the basic decision criterion.

Scope of the Project

In view of the scarcity problem of potable water, it is the intension of this project work to delve into the history of Asejire Dam and the history of water supply in Ibadan. The objective includes the history of the dam, capacity, and storage and distribution network and to study the observable or discernable problems of water supply and proffer solution to water issues.

The increase in human population in Ibadan has caused a critical increase in the water demand for domestic, industrial, agricultural and other uses. In addition, the celebration in human land use, resulting in the description of the natural hydrologic cycle and the self-purification process, has added a new dimension to the global problem of water supply of water by balancing all the hydrological constraints present.

The annual Safe Yield for Asejire reservoir is estimated by the possible abstraction rate 90% or 95% probability which shows that the Safe Yield of the Asejire reservoir is not sufficient to satisfy the estimated water demand of the city of Ibadan now and in the future.

General Background of the Study Area

Asejire reservoir, situated on River Osun about 25km North of East of Ibadan are the current main raw water source for Ibadan water supply. Asejire Dam belongs to the class of large dams by the standard of International Committee on Large Dam (I COLD).

The dam is an earth-embankment, rock-faced type and contains approximately 400,000 cubic yards of earth fill and rock. The dam crest is at elevation 526, providing 13 feet of freeboard the top pf the gates elevation of 513.

Methodology

Optimizing of water resources distribution systems has gained much attention and mathematical programming techniques are one of the many tools available to determining optimal configuration of a particular process when constrained by the different hydrological constraints affecting the water supply system. A network linear programming finds the minimum cost flows through a network of unidirectional arcs; which comprises transportation and energy costs.

The aim of the water distribution network design is to find the optimal pipe diameter for each nipe in the network for a given layout, demand loading conditions, and an operation policy. The model selects the optimal pipe sizes in the final network satisfying all implicit constraints (e.g. conservations of mass and energy), and explicit constraints (e.g. pressure head and design constraints). The hydraulic constraints, for example deal with hydraulic head at certain nodes to meet a specified minimum value. If the hydraulic head constraints is violated, the penalty cost is added to the network cost. However, diameter constraints enforce the evolutionary algorithms to select the trail solution within a predefined limit. A hydraulic network handles the implicit constraints and simultaneously evaluates the hydraulic ‘ performance of each trail solution that is a. member of population of point. The fairness of a trail solution representing a pipe network design is based on the hydraulic performance of the network.

Water Assignment

Over a given season (which may be one or more months), water supply planning model assigns water according to the criteria listed below:

1. Satisfy all demand (which may be restricted).
2. Satisfy in-stream requirement.
3. Minimizing spill from reservoirs and gravitydiversions.
4. Ensuring that water assignments areconsistent with user-defined operating rules.
5. minimize operating costs.

The water supply for the provision of water for domestic, industrial and other uses takes into consideration the demand anticipated for such system. The objective of the supply model as being to determine the optimal scheduling of water resources projective construction (or capacity expansion) in order to satisfy given demands (for freshwater; irrigation water and power) at a minimum cost.

Design and Formulation

Node-arc asmenclature is used in developing the mixed integer linear programming model. A node is some point of interest such as a junction of two streams, a reservoir, a demand, and a diversion point and so on. An arc is simply a hydraulic connection between the nodes.

Objective Function

A linear programming is driven by the objective function. There are two types of objective notions that may be used in this formulation.. The first is a physically-based objective, such as minimize the flow leaving the water supply system. The second is a priority-based mechanism.

According to work done by Chon (1978),on the Alameda County Water District Supply in San Francisco, United State of America, the priorities used in the model are intended to accomplish the following monthly operations, in descending order:

1. Meet all demands
2. Recharge the ground water as much as possible
3. Keep water in surface reservoir storage
4. Use surface supplies rather than drawing down aquifers
5. Minimize spill
6. Minimize pumping and treatment cost.

Constraint Function

The operational described in the objective function formulation are used to form the constraints set. The constraints used by the linear programming are relatively a simple one which includes the following:

1. Continuity at each node
2. Deliveries limited to demands
3. Reservoir storages within acceptable limits
4. Flows through treatment and blending plants limited to plant capacity
5. Meet minimum flow requirements, if any

Model Formulation

Linear and mixed-integer programming are models that are most frequently encountered for planning and scheduling. The reason is that these models involve in most cases discrete time representations coupled with fairly simple performance models. While most models were linear progrramme most of them are now mixed-integers-linear programmes due to the discrete decisions that are involved in investment, expansion and operation for planning, and assignment and sequencing decisions for scheduling.

Mixed-integer Linear Programming (MILP) problems have the general form:

$$\text{Min } Z = a^T x + b^T y$$

Constraints Ax + by d

X 0, y € [0,l]m

When no discrete variables y are involve , the problem reduces to a Linear Programming Problem. This is a special class of convex optimization problem for which the optimal solution lies at a vertex of the poly-type defined by the inequalities Ax d. The solution to Linear Programming problems relies largely on the simplex algorithm.

To develop the Mixed-Integer Linear Programming (MILP) model Kessler and Shamir (1989), concludes that the model's objective is to prescribe the homes of operation for each pump or pump combination whether power is purchased commercially or generated on-site. The optimal ending storage levels in the raw water reservoir each time period are considered.

The total energy costs are to be minimized. Physical constraints on the model are the satisfaction of demands and minimum and maximum reservoir storage.

The complete objective function is given as in equation 2.1 including energy charges, and generator operating cost.

BOP BP BOP BP

Minimize BC(Ecj) (kwixTcij + GCix)

J=1 i=l j=l i=l

$KWi\ xTgij + Z^1 + Z^2$

Subject to the constraints:

BOP
(Tcij+Tgij) TOTHR 2.2
i=j .... BOP

Tcij + Tgij MINij 2.3

For **1**= 1, ...BP
j= 1, ...BOP

SK - SK - 1 - (Qik (Tcij + TGij) <u>(HRk)</u> DkxHRk 2.4
i=j TOTHRi

for k = 1,.................................... BPTOU Qik

SMINk Sk SMAXk 2.5

For K = 1,....................BPTOU
2.6
DCON x Kwi x ij - Z1 0............

For i = 1,..............................BP

j = 1,.............................BOP
j = on - peak
DCOFF x KW x ij -Z,^Z**2** = 0 . 2.7
DCON

For i =1,..................BP
j =1,................BOP
j = off - Peak

where TOTHRj = Total number of hones in operating periods j Q
= Discharge
MINij = Minimum run time in hones for pump i during operating periods j
BC = Energy Costs
BPTOU= The number of time - of - use periods in the cycle period SMINk and
SMAXk = The minimum and maximum ending storage
KW = Energy Power Cost TC Time use of Electricity
GC Generator Cost
TG Time of Generating

DCOF = The off-peak, excess, unit demand and charge
BOP = Number of period of operation
BP = Number of period
DCON = The on-peak, excess, unit demand charge
Ec = Energy cost of pump
Dk Demand

Working Mechanism of Mixed-Integer-Linear Proramming (MILP) Algorithm

A brief description of the steps in the MILP algorithms for the energy cost optimization of Asejire Dam is given as follows and flow chart is shown in figure 1.

- Generate N population of points randomly on the solution space. Each of the population represents a possible combination of storage cycle.
- Compute the operation and energy cost for each of the N solution after converting the randomly generated energy cost.
- Perform hydraulic analysis of each cost by using simplex algorithms to solve equations generated.
- Compute energy cost for the combination of operation period and reservoir storage.
- Select optimal solution that gives minimum energy cost with a combination of optimal constraints.

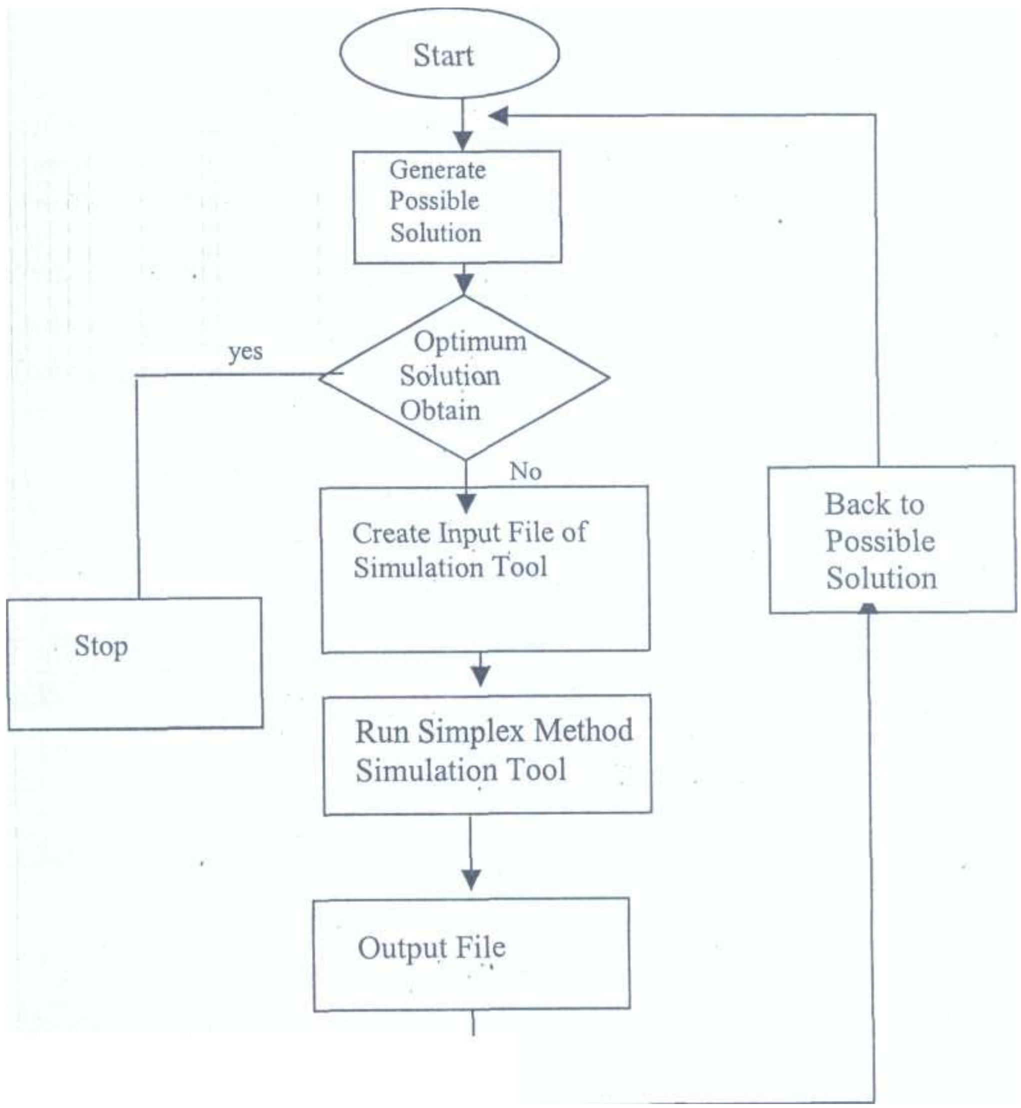

Figure 1: Flow Chart of the Design Problem

Analysis

The combination of the Mixed-Integer Linear Programming (MILP) model developed, with the assumed linearised constraints yields a large non-convex solution area. For simplicity the energy charged from Electric Power Authority and generator operating cost. These costs are used in collaboration with the storage level in the clear well based on specified discharge pressure, and the operated at designated run times.

Data Application to the Model

The objectives of this application is to simulate the cost that would have been incurred had the system been operated in accordance with the doctrine of Mixed-Integer Linear Programming (MILP) model, recommendations and then comparing these costs with those in use at the Dam at present.

In applying the model to obtain optimized solutions foe the problem, use of simplex method of solving for the optimized solutions for the linear programming model was applied. Table 1 which is T.D.H. chart for the low lift pumps shows, the values of Discharging Pressure for different values of storage levels in the dam. Also, the current values of energy charges in use at Asejire Dam are shown in Table 2 and in the dam. These values are then used as input data which represent constraints for the Mixed-Integer Linear Programming (MILP) model shown in equation 3.1.

Water storage level				Discharge Pressure Gage Reading													
	124	122	120	118	116	114	112	110	108	106	104	102	100	98	96	94	92	90
515	140	138	136	134	132	130	128	126	124	122	120	118	116	114	112	100	108	106
514	141	139	137	135	133	131	129	127	125	123	121	119	117	115	113	111	109	107
513	142	140	138	136	134	132	130	128	126	124	122	120	118	116	114	112	110	108
512	143	141	139	137	135	133	131	129	127	125	123	121	119	117	115	113	111	109
511	144	142	140	138	136	134	132	130	128	126	124	122	120	118	116	114	112	110
510	145	143	141	139	137	135	133	131	129	127	125	123	121	119	117	115	113	111
509	146	144	142	140	138	136	134	' 132	130	128	126	124	122	120	118	116	114	112
508	147	145	143	141	139	137	135	133	131	129	127	125	123	121	119	117	115	113
507	148	146	144	142	140	138	136	134	132	130	128	126	124	122	120	118	116	114
506	149	147	145	143	141	139	137	135	133	131	129	127	125	123	121	119	117	115
505	150	148	146	144	142	140	138	136	134	132	130	128	126	124	122	120	118	116
504	151	149	147	145	143	141	139	137	135	133	131	129	127	125	123	121	119	117
503	152	150	148	146	144	142	140	138	136	134	132	130	128	126	124	122	120	118
502	153	151	149	147	145	143	141	139	137	135	133	131	129	127	125	123	121	119
501	154	152	150	148	146	144	142	141	138	136	134	132	130	128	126	124	122	120
500	155	153	151	149	147	145	143	141	139	137	135	133	131	129	127	125	123	121

Table 2: Operating Energy Cost Data for pumps used and Run Time. (Asejire Dam)

Storage Level (meters)	Discharge Pressure	Run Time (hours)	Present Energy Cost (x 1000 Naira)
515	140	3	600
514	140	3	600
513	142	3	600
512	143	4	550
511	144	3	550
510	145	3	550
509	146	3	500

water

508	147	3	500
507	148	3	450
506	149	3	450
505	150	3	450
504	151	3	400
503	152	3	400
502	153	3	360
501	154	3	360
j 000	155	3	360

NOP NP

Minimize BC (S ECj EKWiXTCij + E EGCi x Kwi x TGij) + Z_x + Z_2 ....3.1

j=l i=l i=l

Subject to the constraints:

NOP

S TCij + TGij) > TOTHR i=j

..................................NOP

TCij + TGij MIN

i = 1,................. NP

j=l .... .NOP

SK - SK - (Qik (TCij + TGij) (HRK) Dk x HRk

TOTHRj

For K= 1,................ ....NPTOU

SMNIk Sk SMAXk

For K = 1,.............. ....NPTOU

DCON x Kwix i ij - Z, 0

For i = 1,................ ...NP

j=i, ...NOP

j on peak

DCOFF x Kwi x i ij — Zi + z? c

DCON"

For i = 1,................ .NP

j =E................ NOP

j off peak

Computation for the Mixed-Integer Lihear Programming (MILP) Model

All the feasible solution to the objective function to reduce energy costs, as obtained from Electrical Power Authority and that obtained from operation of Generator can be developed by solving the function with the constraints functions. In Mixed-Integer Linear Programming, are typically used to find lower bounds on the objective that allow one to decide whether a given node in

the branch tree can be obtained. For reliable results, it is therefore imperative that the computed lower bound is rigorously valid.

In solving for the feasible solutions to the objective function, Simplex method **of** solving linear programming problems was employed in order to implement a computational method **of** analysis of the problem.

Table 3: Result of Computation for Total Energy Costs for Storage Levels and Run-Time Values (Asejire Dam)

Storage Level (Meters)	Discharge Pressure	Run-time (hours)	Computed Energy Costs (xlOOO Naira)
515	140	3	450
514	141	3	443
513	142	3	440
512	143	4	433
511	144	3	400
510	145	3	345
509	146	3	340
508	147	3	325
507	148	3	300
506	149	3	270
505	150	3	220
504	151	3	180
503	152	3	130
502	153	3	110
501	154	3	90
500	155	3	80

Solutions to the Mixed-Integer Linear Programming Problem

As shown in Table 3 the solution to the model for the feasible solutions tot the Mixed-Integer Linear problem obtained after the iteration procedure performed by the use of the Fortran programme developed for the problem iteration reveals that the total Energy cost reduces progressively and considerably as the storage level increases.

In the solution result comprise of 16 values of run-time after the iteration was performed at **10** runs of iteration level in the Fortran programme. The minimum level of storage values and that of discharge pressure are given at 500meter§ and 140 respectively were used for each node. The iteration runs were performed using different initial seed value. It can be seen that the optimal solution obtained from each seed values satisfies pressure constraints applied to all nodes. Table 3 lists the optimal solution; total energy costs, storage level, pressure values, and number of the run-time. It can be deduced from the results obtained that there is a maximum of 25% decrease in operating energy costs when the Mixed-Integer Linear Programming model was used.

Recommendations

The Mixed-Integer Linear Programming (MILP) models has been used occasionally to support general planning decisions, but its full implementation must be planned until the hydrologic models have demonstrated enough predictive accuracy, based on recent records, to satisfy both operators and planners.

It is recommended that other possible variables should be considered when developing the Mixed-Integer Linear Programming model for water system; such variables include pipe diameter and pipe roughen.

Conclusion

The results of the study present an overview of planning and scheduling. It has been shown that the problem of optimizing energy costs for the analysis of the performance of the Asejire dam

leads to discrete optimization models for which the associated mathematical programming problems corresponding to a mixed integer linear programming problems. The use of the Simplex method of analyzing linear programming problems offers the potential of not only simplifying the formulations, but also of decreasing the computational requirements.

It is clear that the use of the Mixed-Integer Linear Programming optimization model offer an effective means of dealing with the problems of optimally sequencing energy costs with the storage level in the reservoir.

The solution obtained reveals a considerable reduction in the total energy cost as obtained from electricity supply and from generating the energy. It s shown that the maximum value of 600.0 Naira operating energy cost as a storage level of 515meters was reduced to a value of 450.0 Naira energy cost when the MILP model was used; this represents a 25% reduction in the energy cost due to the application of the'MILP model.

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